Bell work

1. There are 5 finalists in the Mr. Rock Hill pageant. In how many ways may the judges choose a winner and a first runner-up?

$\underset{W}{5} \cdot \underset{R\text{-}U}{4} = 20$

2. A multiple choice test consists of 15 questions, each permitting a choice of 5 alternatives. In how many ways may a student fill in the answers if they answer each question?

$\underset{1}{5} \cdot \underset{2}{5} \cdot \underset{3}{5} \cdots\cdots \underset{15}{5}$ $\quad 5^{15} = 3.05 \times 10^{10}$

3. How many distinguishable permutations are there of the letters in the word "gr~~ee~~t"?

$\frac{5!}{2!} = 60$

Combinations: Order doesn't matter

$_nC_r = \frac{n!}{(n-r)!r!}$ $\quad {}_nP_r = \frac{n!}{(n-r)!}$

How can you choose 5 cards from a 52-card deck?

$_{52}C_5 = 2{,}598{,}960$

How can you choose 5 cards the same color from a 52-card deck?

$_2C_1 \cdot {}_{26}C_5 = 131{,}516$

When working with events:

Both: A and B Multiply the events

One or the other: A or B Add the events

William Shakespeare wrote 38 plays that can be divided into 3 genres. Of the 38 plays there are 18 comedies, 10 histories, 10 tragedies.

How many different sets of exactly 2 comedies and 1 tragedy can you read?

$_{18}C_2 \cdot {}_{10}C_1 = 1{,}530$

How many different sets of exactly 3 tragedies or 2 histories can you read?

$_{10}C_3 + {}_{10}C_2 = 165$

How many different sets of at most 3 plays can you read?

0 plays $_{38}C_0 = 1$

+

1 play $_{38}C_1 = 38$

+

2 plays $_{38}C_2 = 703$

+

3 plays $_{38}C_3 = 8436$

9178

During the school year, the girl's basketball team is scheduled to play 12 home games. You want to attend at least 3 of the games. How many different combinations of games can you attend?

3 games $_{12}C_3 = 220$
4 games $_{12}C_4 = 495$
5 games $_{12}C_5 = 792$
6 games $_{12}C_6 = 924$
7 games $_{12}C_7 = 792$
8 games $_{12}C_8 = 495$
9 games $_{12}C_9 = 220$
10 games $_{12}C_{10} = 66$
11 games $_{12}C_{11} = 12$
12 games $_{12}C_{12} = 1$

4,017

total combinations − combinations not using

$\frac{2}{1}\ \frac{2}{2}\ \frac{2}{3} \cdots\cdots \frac{2}{12}$

2 games $_{12}C_2$
1 game $_{12}C_1$
0 game $_{12}C_0$

$2^{12} - ({}_{12}C_2 + {}_{12}C_1 + {}_{12}C_0)$

Find the number of possible 5-card hand that contain the cards specified. The cards are taken from a standard 52-card deck.

1) 5 face cards
$_{12}C_5 = 792$

2) 4 kings and 1 other card
$_4C_4 \cdot {}_{48}C_1 = 48$

3) 1 Ace and 4 not aces
$_4C_1 \cdot {}_{48}C_4 = 778{,}320$

4) 5 hearts or 5 diamonds
$_{13}C_5 + {}_{13}C_5 = 2{,}574$

5) At most 1 Queen out of the 5 cards
0 Queen $_4C_0 \cdot {}_{48}C_5 = 1{,}712{,}304$
1 Queen $_4C_1 \cdot {}_{48}C_4 = 778{,}320$
$+$
2,490,624

6) At least 1 spade
1 spade $_{13}C_1 \cdot {}_{39}C_4 = 1{,}069{,}263$
2 spade $_{13}C_2 \cdot {}_{39}C_3 = 712{,}842$
3 spade $_{13}C_3 \cdot {}_{39}C_2 = 211{,}926$
4 spade $_{13}C_4 \cdot {}_{39}C_1 = 27{,}885$
5 spade $_{13}C_5 \cdot {}_{39}C_0 = 1{,}287$
2,023,203

total combinations − not using

$_{52}C_5 - {}_{13}C_0 \cdot {}_{39}C_5$