Conditional Probability

for dependent events

formula for conditional probability:

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

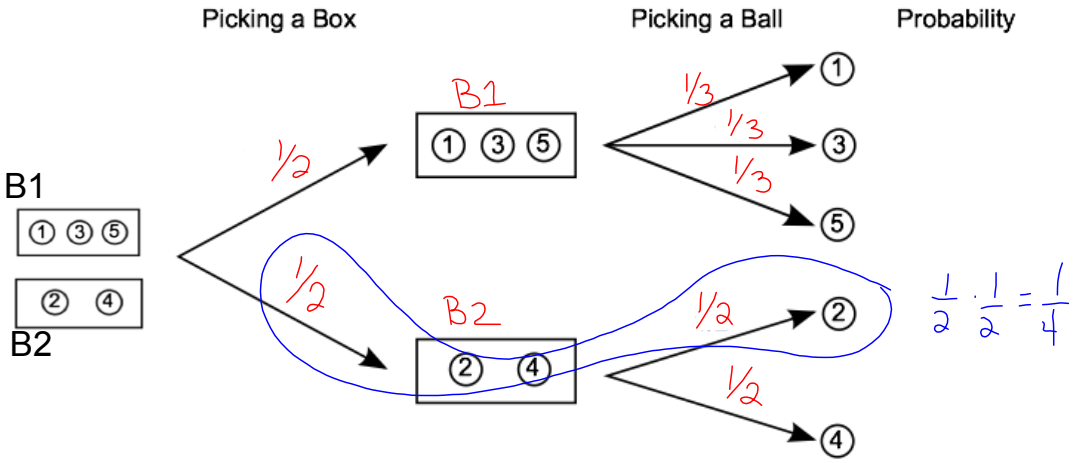
"Probability of B given A"

Write the formula for the "Probability of A given B".

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

You are given two boxes with balls numbered 1-5. One contains balls 1,2,3 and the other contains 2,4. You first pick a box at random, and then you select a ball at random. What is the probability of picking a 2?

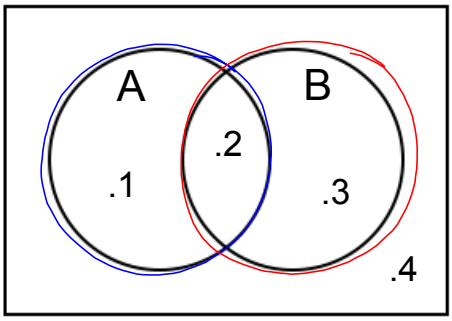
picking a 5? $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$



P(3|B1)= 1/3

P(2|B2)= 1/2

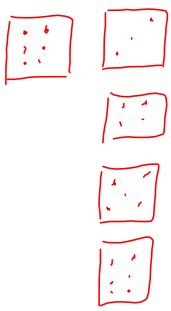
P(5 | B2) = 0



$$P(B \mid A) = \frac{.2}{.3} = \frac{2}{3}$$

$$P(A \mid B) = \frac{.2}{.5} = \frac{2}{5}$$

What is the probability that the sum of two die will be greater than 8, given that the first die is 6?



$$\frac{4}{6} = \frac{2}{3}$$

events A and B are independent
if and only if they satisfy

$$P(B \mid A) = P(B) \quad \text{or} \quad P(A \mid B) = P(A)$$

6) A coin is tossed and a single 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.

Event A

$\frac{P(A \cap B)}{P(A)} = ?$

Event B

$\frac{P(A \cap B)}{P(B)} = ?$

Are the two events independent or dependent?
How do you know?

$\frac{1/2 \cdot 1/6 = 1/12}{1/2} = \frac{1}{6}$ $P(3) = 1/6$

Key:
Male = M Female = F
Blue = B Not Blue = N

Sample size = 200

P(B) = $\frac{84}{200}$ P(Blue)

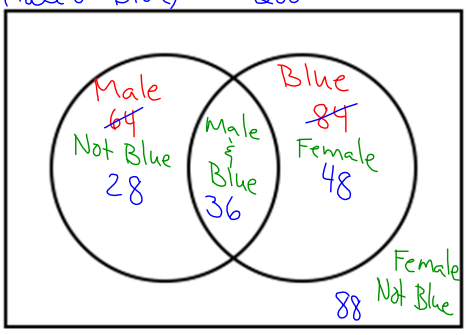
P(M) = $\frac{64}{200}$ P(Male)

P(F|B) = $\frac{48}{84}$ P(Female | Blue)

P(B|F) = P(Blue | Female) $\frac{48}{136}$

P(M∩B) = P(Male & Blue) = $\frac{36}{200}$

P(M∪B) = P(Male or Blue) = $\frac{112}{200}$



	Blue	Not Blue	Total
Male	36	28	64
Female	48	88	136
Total	84	116	200

Is color preference independent of gender?
How do you know?

P(A) = Male
P(B) = Blue

$P(A \mid B) = P(A)$

$\frac{P(A \text{ and } B)}{P(B)} = P(A)$

$\frac{36}{84} = \frac{64}{200}$

.43 ≠ .32