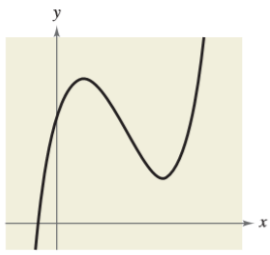
Chapter 2 day 2

- Use transformations to sketch graphs of polynomial functions.
- Use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions.
- Find and use the real zeros of polynomial functions as sketching aids.
- Use the Intermediate Value Theorem to help locate real zeros of polynomial functions.

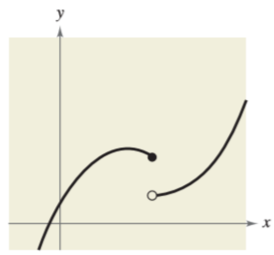
2 Features of a polynomial function.

*The first feature is that the graph of a polynomial function is **continuous.**

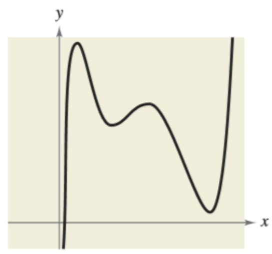
*The second feature is that the graph of a polynomial function has only smooth, rounded turns.



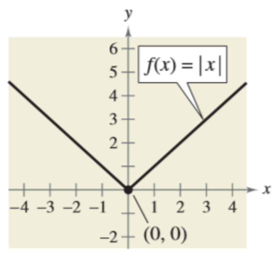
(a) Polynomial functions have continuous graphs.



(b) Functions with graphs that are not continuous are not polynomial functions.



Polynomial functions have graphs with smooth, rounded turns.

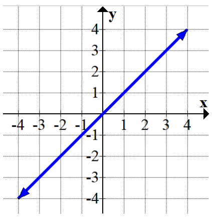


Graphs of polynomial functions cannot have sharp turns.

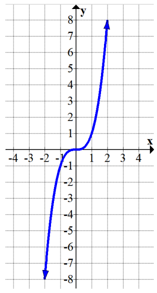
Graphs of Polynomial Functions

Using the features presented in this section, coupled with your knowledge of point plotting, intercepts, and symmetry, you should be able to make reasonably accurate sketches *by hand*.

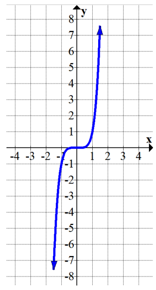
The polynomial functions that have the simplest graphs are monomials of the form $f(x) = x^n$, where n is an integer greater than zero.



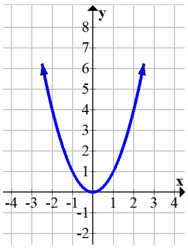
$y = x^1$



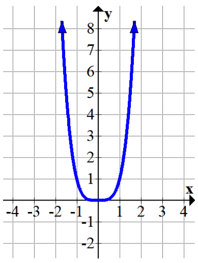
$y = x^3$



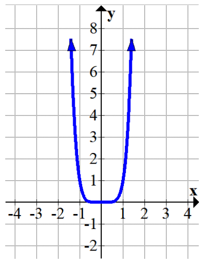
$y = x^5$



$y = x^2$

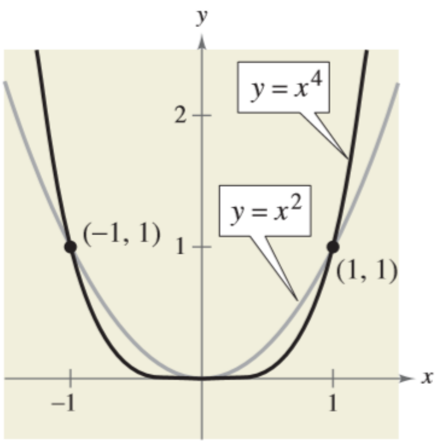


$y = x^4$

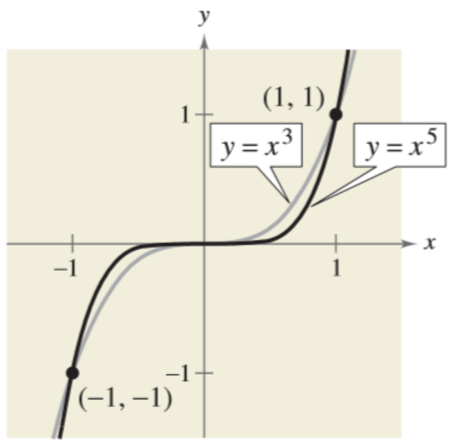


$y = x^6$

Polynomial functions of the form $f(x) = x^n$ are often referred to as power functions.



(a) If n is even, the graph of $y = x^n$ touches the axis at the x-intercept.



(b) If n is odd, the graph of $y = x^n$ crosses the axis at the x-intercept.

The greater the value of n, the flatter the graph near the origin.

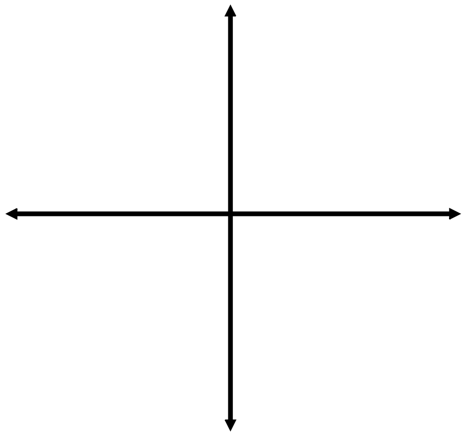
Sketching Transformations of Monomial Functions

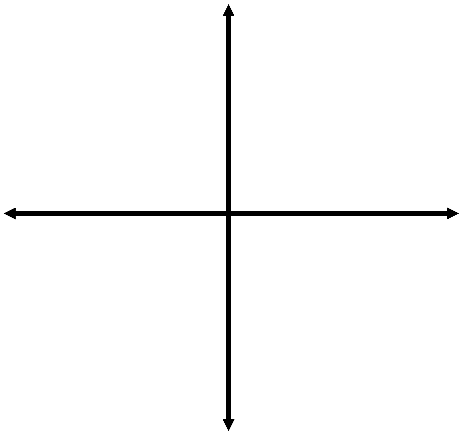
Sketch the graph of each function.

a) $f(x) = -x^5$

b) $h(x) = (x+1)^4$

Pull

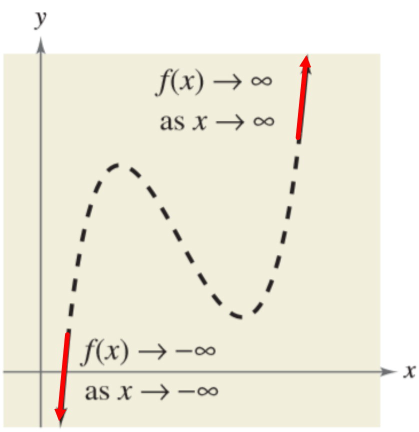




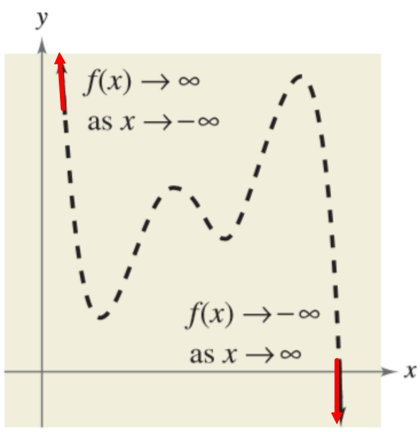
Pull

The Leading Coefficient Test

When n is odd:



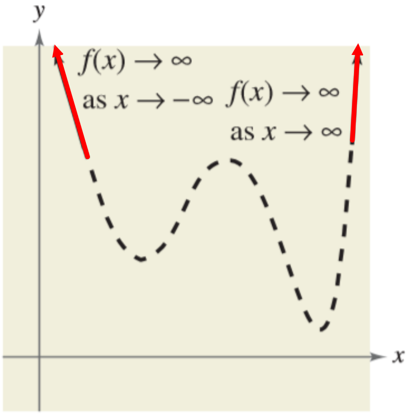
a is (+)



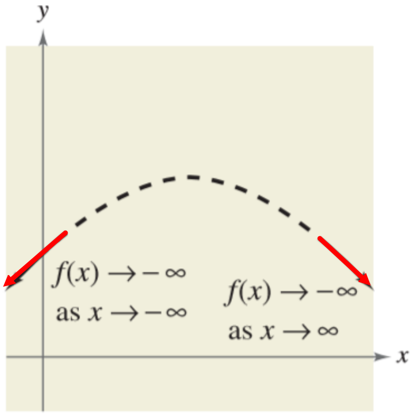
a is (-)

The Leading Coefficient Test

When n is even:



a is (+)



a is (-)

State the end behavior without using a calculator.

a) $f(x) = -x^3 + 4x$ b) $f(x) = x^4 - 5x^2 + 4$ c) $f(x) = x^5 - x$

Pull

Zeros of Polynomial Functions

1. The function f has, at most n, real zeros.

2. The graph of f has, at most, n - 1 turning points. (Turning points, also called relative minima or relative maxima, are points at which the graph changes from increasing to decreasing or vice versa.)

Real Zeros of Polynomial Functions

If f is a polynomial function and a is a real number, the following statements are equivalent.

1. $x = a$ is a *zero* of the function f.

2. $x = a$ is a *solution* of the polynomial equation $f(x) = 0$.

3. $(x - a)$ is a *factor* of the polynomial $f(x)$.

4. $(a, 0)$ is an *x-intercept* of the graph of f.

$f(x) = -2x^4 + 2x^2$

Possible Zeros:

Maximum possible turning points?

Find all Real Zeros:

$f(x) = 3x^4 + 9x^2 + 6$

Possible Zeros:

Maximum possible turning points?

Find all Real Zeros:

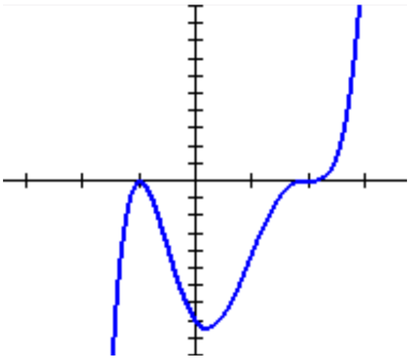
Repeated Zeros

A factor $(x - a)^k$, $k > 1$, yields a **repeated zero** $x = a$ of **multiplicity** k.

1. If k is odd, the graph *crosses* the x-axis at $x = a$.

2. If k is even, the graph *touches* the x-axis (but does not cross the x-axis) at $x = a$.

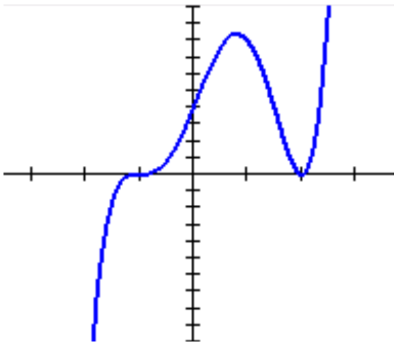
$f(x) = (x - 2)^3(x + 1)^2$



Zeros:

Multiplicity:

$f(x) = (x + 1)^3(x - 2)^2$



Zeros:

Multiplicity:

Sketch the graph of :

1. Apply the leading Coefficient Test
2. Find the Zeros
3. Plot extra points if needed

$f(x) = 3x^4 - 4x^3$

$f(x) = -2x^3 + 6x^2 -(9/2)x$

What if the function doesn't factor? How can we find a zero without our calculator?

1. Apply the leading Coefficient Te
2. Find the Zeros
3. Plot extra points if needed

$$f(x) = x^3 + x^2 + 1$$

$f(-2) =$

$f(-1) =$

$f(0) =$

$f(1) =$

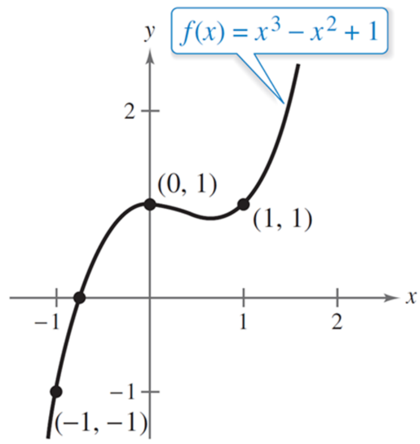
Pull

Intermediate Value Theorem

Let a and b be real numbers such that $a < b$. If f is a polynomial function such that $f(a) \neq f(b)$, then, in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.

How accurate can you get on the zeros?

$$f(x) = x^3 - x^2 + 1$$



Write the polynomial with the give zeros.

$x = -2, 1$

$x = 5, -2, 2, 1$

Write the polynomial with the give zeros and the given degree.

$x = 2, 5, 6$ and $n = 4$

More than one correct answer

Section 2.2

Pg. 132-135: #9-14, 16, 17, 19-27 odd, 34, 40, 43, 46, 54, 55, 57, 67, 81, 95, 106