

$$46) f(t) = 2t^4 - 2t^2 - 40$$

Max turn: 3 Max zero: 4

$$0 = 2(t^4 - t^2 - 20)$$

$$t = \pm\sqrt{5}, \pm 2i$$

$\pm\sqrt{5}$ odd (1)

$-\sqrt{5}$ odd (1)

$\pm 2i$ odd (1)

$-2i$ odd (1)

$$0 = 2(t^2 - 5)(t^2 + 4)$$

$$t^2 - 5 = 0 \quad t^2 = -4$$

$$t^2 = 5 \quad t = \pm\sqrt{-4}$$

$$t = \pm\sqrt{5} \quad t = \pm 2i$$

$$43) g(t) = t^5 - 6t^3 + 9t$$

Max zero: 5

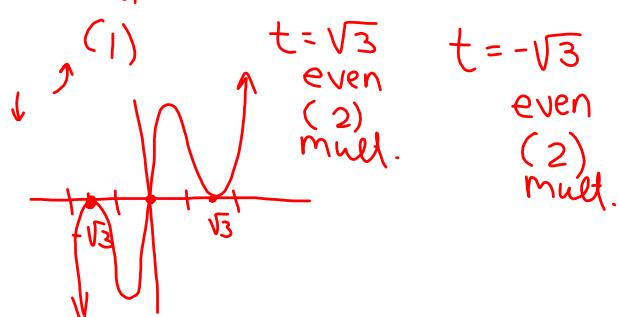
Max turn: 4

$$0 = t(t^4 - 6t^2 + 9)$$

$$0 = t(t^2 - 3)(t^2 - 3)$$

$$t = 0 \quad t^2 = 3$$

$$\begin{array}{ll} \text{mult.} & t = \pm\sqrt{3} \\ \text{odd} & t = \pm\sqrt{3} \end{array}$$

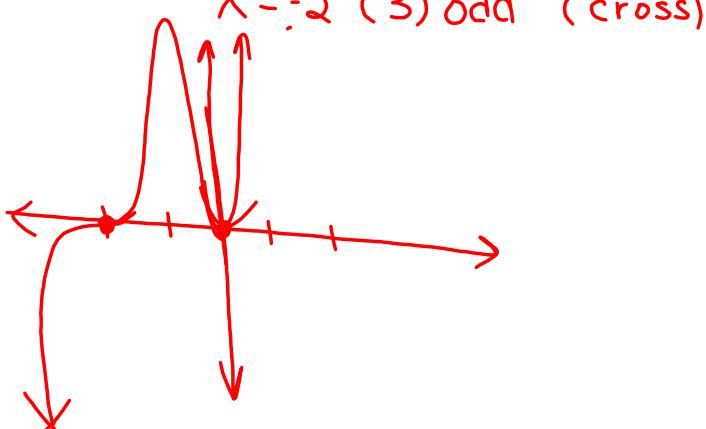


$$81) \quad f(x) = 9x^2(x+2)^3$$

Degree: 5
Turn: 4

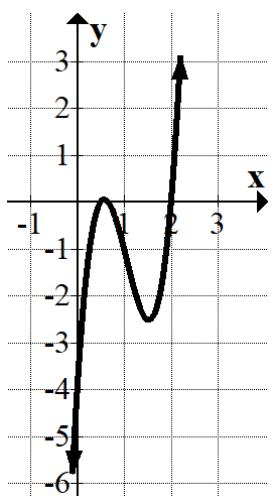
Zero: $x=0$ (2) even (touch)

$x=-2$ (3) odd (cross)



2.3A Notes

$$f(x) = 6x^3 - 19x^2 + 15x - 4$$



Long Division

Notice that one of the zeros of f is $x = 2$. This means that $(x - 2)$ is a factor of $f(x)$ and there exists a second-degree polynomial $q(x)$ such that:

$$\begin{aligned} f(x) &= (x - 2) \cdot q(x) \\ &= (x - 2)(6x^2 - 7x + 1) - \frac{2}{x-2} \underline{- 6x^3 + 12x^2} \\ &\quad \underline{\underline{- 7x^2 + 15x}} \\ &\quad \underline{\underline{+ 7x^2 - 14x}} \\ &\quad \underline{\underline{- X + 4}} \\ &\quad \underline{\underline{- X + 2}} \end{aligned}$$

How do you find $q(x)$?

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, there exist unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x)q(x) + r(x)$$

↑ ↑ ↑ ↑
Dividend Divisor Quotient Remainder

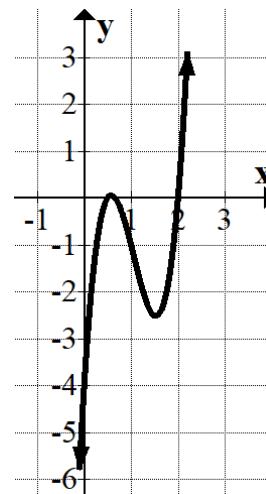
where $r(x) = 0$ or the degree of $r(x)$ is less than the degree of $d(x)$. If the remainder $r(x)$ is zero, $d(x)$ divides evenly into $f(x)$.

Another way to write the division algorithm is:

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

$$f(x) = 6x^3 - 19x^2 + 15x - 4$$

$$d(x) = x - 2$$



Find all of the zeros by long division.

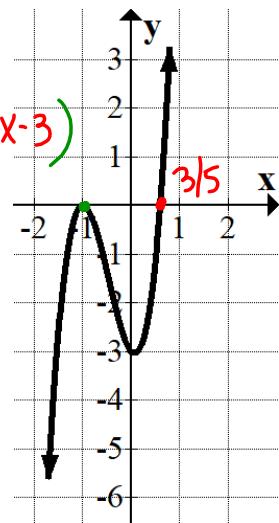
$$\text{divisor} \Rightarrow \frac{x+1}{x^2+2x+1}$$

$$f(x) = 5x^3 + 7x^2 - x - 3$$

$$(x+1)(x+1)(5x-3)$$

Pull

$$\begin{array}{r} x^2 + 2x + 1) \overline{) 5x^3 + 7x^2 - x - 3} \\ \underline{- 5x^3 - 10x^2 - 5x} \\ \underline{\underline{- 3x^2 - 6x - 3}} \\ \underline{\underline{+ 3x^2 + 6x + 3}} \\ 0 \end{array}$$



Divide by long division:

$$f(x) = x^3 - 1 \quad d(x) = x - 1$$

$$\begin{array}{r} x^2 + x + 1 \\ \hline x-1) x^3 + 0x^2 + 0x - 1 \\ \underline{- x^3 + x^2} \\ \underline{\underline{x^2 + 0x}} \\ \underline{- x^2 - x} \\ \underline{\underline{x - 1}} \\ \underline{- x + 1} \\ 0 \end{array}$$

$$(x-1)(x^2+x+1) = x^3 - 1$$

Divide by long division:

$$f(x) = x^3 - 2x^2 - 9 \quad d(x) = x - 3$$

$$\begin{array}{r} x^2 + x + 3 \\ \hline x-3) x^3 - 2x^2 + 0x - 9 \end{array}$$

Divide by long division:

$$f(x) = 2x^4 + 4x^3 - 5x^2 + 3x - 2 \quad d(x) = x^2 + 2x - 3$$

$$\begin{array}{r} 2x^2 + 1 \\ \hline x^2 + 2x - 3) 2x^4 + 4x^3 - 5x^2 + 3x - 2 \\ - 2x^4 + 4x^3 + 6x^2 \quad \downarrow \quad \downarrow \\ \hline x^2 + 3x - 2 \\ - x^2 - 2x - 3 \\ \hline x + 1 \end{array} \quad + \frac{x+1}{x^2+2x-3} \quad x \neq -3, 1$$

$(x^2 + 2x - 3)(2x^2 + 1) + \frac{x+1}{x^2+2x-3}$

Write the function in the form $f(x) = (x - k)q(x) + r$ for the given value of k ,
 and demonstrate that $f(k) = r$. (remainder theorem)

$$f(x) = 15x^4 + 10x^3 - 6x^2 + 14, k = -2/3$$

$$\frac{(3x+2)(5x^3 - 2x^2 + 4/3) + \frac{34}{3}}{3x+2}$$

$$f(-2/3) = 15(-2/3)^4 + 10(-2/3)^3 - 6(-2/3)^2 + 14$$

$$f(-2/3) = \frac{34}{3}$$

divisor
 $K = -2/3$

$$x = -2/3 \quad (3x+2)$$

$$\begin{array}{r} & 5x^3 & -2x & +4/3 \\ \hline 3x+2) & 15x^4 & +10x^3 & -6x^2 & +0x +14 \\ & -15x^4 & -10x^3 & & \\ \hline & 0 & +0 & -6x^2 & +0x \\ & & & +6x^2 & +4x \\ \hline & & & 4x & +14 \\ & & & -4x & -\frac{8}{3} \\ \hline & & & \frac{34}{3} & \\ \end{array} \quad \left(\frac{4}{3}\right)\left(2\right) \quad 14 - \frac{8}{3}$$

Write the function in the form $f(x) = (x - k)q(x) + r$ for the given value of k ,
 and demonstrate that $f(k) = r$. (remainder theorem)

$$f(x) = x^3 - x^2 - 10x + 7, k = -2/3$$

$$\frac{349}{27}$$

$$\begin{array}{r} & \frac{1}{3}x^2 & -\frac{5}{9}x & -\frac{80}{27} \\ \hline 3x+2) & x^3 & -x^2 & -10x & +7 \\ & -x^3 & -\frac{2}{3}x^2 & & \\ \hline & & -\frac{5}{3}x^2 & -10x & \\ & & +\frac{5}{3}x^2 & +\frac{10}{9}x & \\ \hline & & -\frac{80}{9}x & +7 & \\ & & +\frac{80}{9}x & +\frac{160}{27} & \\ \hline & & \frac{349}{27} & & \\ \end{array}$$

$$\frac{160}{27} + 7$$

Section 2.3 day 1

Pg. 142-144: #1, ~~X~~, 8, 10, 13, ~~X~~, 21, 22, ~~X~~, 46, 48, 79, 83-86, 90