Section 2.5 A Synthetic Division Rational Roots

- Use the Fundamental Theorem of Algebra to determine the number of zeros of polynomial functions.
- Find rational zeros of polynomial functions.
- Find zeros of polynomials by factoring.

The Fundamental Theorem of Algebra

If $f(x)$ is a polynomial of degree n, where $n > 0$, then f has at least one zero in the complex number system.

Linear Factorization Theorem

If $f(x)$ is a polynomial of degree n, where $n > 0$, then f has precisely n linear factors

$$f(x) = a_n(x - c_1)(x - c_2) \cdot \cdot \cdot (x - c_n)$$

where $c_1, c_2, \ldots, c_n$ are complex numbers.

Note that the Fundamental Theorem of Algebra and the Linear Factorization Theorem tell you only that the zeros or factors of a polynomial exist, not how to find them.

How many zeros and what are they?

$f(x) = x - 2$

$f(x) = x^2 - 6x + 9$

$f(x) = x^3 + 4x$

$f(x) = x^4 - 1$

The Rational Zero Test

If the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdot\ \cdot\ \cdot + a_2x^2 + a_1x + a_0$ has *integer* coefficients, every rational zero of f has the form

$$\text{Rational zero} = \frac{p}{q}$$

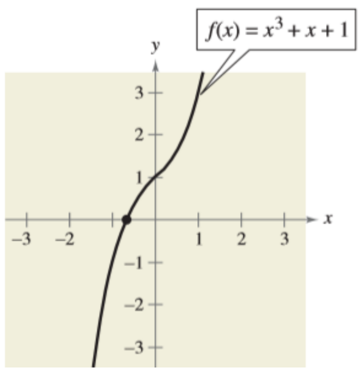
where p and q have no common factors other than 1, and

p = a factor of the constant term a_0

q = a factor of the leading coefficient a_n.

Find the rational zeros.

$f(x) = x^3 + x + 1$



Find the rational zeros of

$f(x) = x^4 - x^3 + x^2 - 3x - 6$

Find all of the real zeros.

Helpful hints:

1) rational zero test: p/q

2) intermediate value theorem: (change in signs of the remainder)

3) upper and lower bound

4) use graphing calculator

Upper and Lower Bound Rules

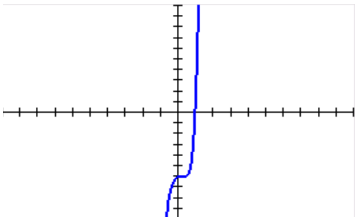
Let $f(x)$ be a polynomial with real coefficients and a positive leading coefficient. Suppose $f(x)$ is divided by $x - c$, using synthetic division.

1. If $c > 0$ and each number in the last row is either positive or zero, c is an **upper bound** for the real zeros of f.
2. If $c < 0$ and the numbers in the last row are alternately positive and negative (zero entries count as positive or negative), c is a **lower bound** for the real zeros of f.

Prove that all of the real zeros lie between the interval [0, 1]

What are the potential rational zeros?

$f(x) = 10x^5 - 3x^2 + x - 6$



Find the real zeros:

$f(x) = x^3 - 15x^2 + 75x - 125$

Find the real zeros:

$$f(x) = 2x^3 + 3x^2 - 8x + 3$$

Find the real zeros:

$$-10x^3 + 15x^2 + 16x - 12 = 0$$

Section 2.5A

Pg. 162-165: #9-12, 17, 18, 22, 27, 29-39 odd