

$$33) \quad f(x) = \frac{Q}{P} x^3 + x^2 - 4x - 4$$

Possible rational zeros

$$\frac{P}{Q}$$

$$\begin{array}{r} \underline{-1} & 1 & 1 & -4 & -4 \\ \downarrow & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$$\pm \frac{1, 2, 4}{1}$$

$$\pm 1, 2, 4$$

$$x^2 - 4 = 0$$

$$(x+2)(x-2) = 0$$

$$x = -2, 2$$

Section 2.5B

Synthetic Division with imaginary numbers

Solve:

$$x^2 + 4 = 0$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$x = \pm \sqrt{-4}$$

$$x = \pm 2i$$

$$x^2 - 3x + 5 = 0$$

$$x = \frac{3 \pm \sqrt{9 - 4(1)(5)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{-11}}{2}$$

$$x = \frac{3 \pm i\sqrt{11}}{2}$$

Imaginary solutions always come in conjugate pairs.

Examples:

$$4i, -4i$$

$$2 + 6i, 2 - 6i$$

$$-5 + 7i, -5 - 7i$$

Write the polynomial with real coefficients that has the given zeros.

Many solutions: what is "a"

$$4, -3i, 3i$$

$$\begin{aligned}f(x) &= a(x-4)(x+3i)(x-3i) \\&= a(x-4)(x^2+9) \\&= a(x^3+9x-4x^2-36) \\&= a(x^3-4x^2+9x-36) \\f(x) &= x^3-4x^2+9x-36\end{aligned}$$

$$-1, 5, 3 - 2i, 3+2i$$

$$\begin{aligned}f(x) &= \underbrace{(x+1)(x-5)}_{(x^2-4x-5)} \underbrace{(x-3+2i)(x-3-2i)}_{\begin{array}{r}x^2-3x-2xi \\ -3x \quad +9+6i \\ +2xi \quad +4-6i\end{array}} \\&= (x^2-4x-5)(x^2-6x+13)\end{aligned}$$

Find the polynomial with real coefficients that has the given degree, zeros, and solution point. "a" is a specific value

Degree: 4 Zeros: $-1, 2, \sqrt{2}i, -\sqrt{2}i$ Point: $f(1) = 12$ find "a"

$$\begin{aligned} f(x) &= a(x+1)(x-2)(x-\sqrt{2}i)(x+\sqrt{2}i) \\ &= a(x^2-x-2)(x^2+i\sqrt{2}x-i\sqrt{2}x+2) \\ &= a(x^2-x-2)(x^2+2) \\ &= a(x^4-x^3-2x^2-2x^2-2x-4) \end{aligned}$$

$$\begin{aligned} f(x) &= a(x^4-x^3-2x-4) \\ |_2 &= a(1-2(1)-4) \\ |_2 &= -6a \\ a &= -2 \end{aligned}$$

$$\begin{aligned} f(x) &= (-2)(x^4-x^3-2x-4) \\ f(x) &= -2x^4+2x^3+4x+8 \end{aligned}$$

Find the polynomial with real coefficients that has the given degree, zeros, and solution point.

Degree: 4 Zeros: $-2, 1 - \sqrt{2}i, 1 + \sqrt{2}i$ Point: $f(-1) = -12$

$$\begin{aligned} f(x) &= a(x+2)(x+2)(x-1+i\sqrt{2})(x-1-i\sqrt{2}) \\ &= a(x^2+4x+4)(x^2-x-i\sqrt{2}x+1+i\sqrt{2}x+2-i\sqrt{2}) \\ &= a(x^2+4x+4)(x^2-2x+3) \\ &= a(x^4-2x^3+3x^2-8x^3+12x+12) \\ &= a(x^4+2x^3-x^2+4x+12) \\ -12 &= a(1-2+1-4+12) \\ -12 &= 6a \\ a &= -2 \\ f(x) &= -2x^4-4x^3+2x^2-8x-24 \end{aligned}$$

Use the given zero to find all of the zeros of the function.

$$\text{Zero: } 3i \quad f(x) = 2x^3 + 3x^2 + 18x + 27 \quad \frac{P}{Q} = \pm \frac{1, 3, 9, 27}{1, 2}$$

$$\begin{array}{r} 3i \boxed{2} & 3 & 18 & 27 \\ \downarrow & 6i & 9i - 18 & -27 \\ \hline -3i \boxed{2} & 3+6i & 9i & 0 \\ \downarrow & -6i & -9i \\ \hline & 2 & 3 & 0 \end{array} \quad x = -\frac{3}{2}$$

$$2x + 3 = 0$$

$$x = -\frac{3}{2}$$

Use the given zero to find all of the zeros of the function.

$$\text{Zero: } 1 - 2i \quad h(x) = 4x^4 + 17x^2 + 14x + 65$$

$$\begin{array}{r} 1-2i \boxed{1} & 4 & 0 & 17 & 14 & 65 \\ \downarrow & 4-8i & -12-16i & & & \\ \hline 1+2i \boxed{1} & 4 & 4-8i & 5+16i & & \\ \downarrow & 4+8i & 8+16i & & & \\ \hline & 4 & 8 & 13 & & \end{array}$$

Write the polynomial as the product of linear factors and list all the zeros of the function.

$$\frac{P}{Q} = \pm 1, 2$$

$$h(x) = x^3 - 3x^2 + 4x - 2$$

$$h(x) = (x-1)(x-1-i)(x-1+i)$$

$$\begin{array}{r} 1 \mid 1 & -3 & 4 & -2 \\ \downarrow & & 1 & -2 & 2 \\ \hline 1 & -2 & 2 & 0 \end{array}$$

$$x^2 - 2x + 2 = 0$$

$$x = \frac{2 \pm \sqrt{4-4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

$|+i|$
 $| -i |$

Write the polynomial as the product of linear factors and list all the zeros of the function.

$$\pm 1, 3, 9$$

$$h(x) = (x+3)^2(x+i)(x-i)$$

$$\begin{array}{r} -3 \mid 1 & 6 & 10 & 6 & 9 \\ \downarrow & -3 & -9 & -3 & -9 \\ -3 \mid 1 & 3 & 1 & 3 & 0 \\ \downarrow & -3 & 0 & -3 & \\ \hline 1 & 0 & 1 & 0 \end{array}$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

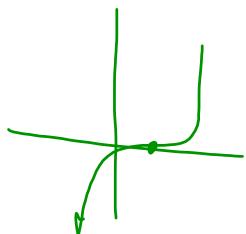
$$x = \pm \sqrt{-1}$$

$$x = \pm i$$

Write the polynomial as the product of linear factors and list all the zeros of the function.

$$h(x) = x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32$$

$$\frac{P}{q} = \pm 1, 2, 4, 8, 16, 32$$



$$\begin{array}{r}
 \begin{array}{c|ccccccccc}
 2 & 1 & 1 & -8 & 28 & -56 & 64 & -32 \\
 & & \downarrow & 2 & -12 & 32 & -48 & 32 \\
 2 & 1 & -6 & 16 & -24 & 16 & -16 & 0 \\
 & & \downarrow & 2 & -8 & 16 & -16 & \\
 2 & 1 & -4 & 8 & -8 & 0 & \\
 & & \downarrow & 2 & -4 & 8 & \\
 & 1 & -2 & 4 & 0 &
 \end{array} \\
 x^2 - 2x + 4 = 0 \quad x = \frac{2 \pm \sqrt{4 - 4(1)(4)}}{2(1)} \\
 x^2 - 2x + 4 = 0 \quad x = \frac{2 \pm \sqrt{-12}}{2} \\
 x = \frac{2 \pm 2i\sqrt{3}}{2} \\
 x = 1 \pm i\sqrt{3}
 \end{array}$$

Section 2.5B

Pg. 162-165: #41, 43, 47, 49, 55, 57, 65, 69, 71, 74, 75, 77, 87, 120