Precalculus
Section 2.6A

Name Key
Period _____ **Date** _____

Find the domain, vertical asymptotes, and horizontal asymptotes for the following rational functions.

1. $f(x) = \dfrac{1}{x-5}$

$(-\infty,5)\cup(5,\infty)$ $(-\infty,0)\cup(0,\infty)$

Domain: $\mathbb{R}, x \neq 5$ Range: $\mathbb{R}\ y \neq 0$

Vertical Asymptote: $x = 5$

Horizontal Asymptote: $y = 0$

2. $f(x) = \dfrac{3x^3+30}{2x^3}$

Domain: $\mathbb{R}, x \neq 0$ Range: $\mathbb{R}, y \neq 3/2$

Vertical Asymptote: $x = 0$

Horizontal Asymptote: $y = 3/2$

3. $f(x) = \dfrac{7x}{2x^2+9x+4}$

$(2x+1)(x+4)$

Domain: $\mathbb{R}, x \neq -\frac{1}{2}, -4$

Vertical Asymptote: $x = -1/2,\ x = -4$

Horizontal Asymptote: $y = 0$

4. $f(x) = \dfrac{2}{x^2-8x}$

$x(x-8)$

Domain: $\mathbb{R}\ x \neq 0, 8$

Vertical Asymptote: $x = 0,\ x = 8$

Horizontal Asymptote: $y = 0$

5. $f(x) = \dfrac{6x^2+3}{x-1}$ $\dfrac{3(2x^2+1)}{(x-1)}$

Domain: $\mathbb{R}\ x \neq 1$

Vertical Asymptote: $x = 1$

Horizontal Asymptote: None

6. $f(x) = \dfrac{5x^3-4}{x^2+4x-5}$

$(x+5)(x-1)$

Domain: $\mathbb{R}\ x \neq -5, 1$

Vertical Asymptote: $x = -5,\ x = 1$

Horizontal Asymptote: None

7. $f(x) = \dfrac{3x^2-1}{x^3}$

Domain: $\mathbb{R}\ x \neq 0$

Vertical Asymptote: $x = 0$

Horizontal Asymptote: $y = 0$

8. $f(x) = \dfrac{2x^2+x-9}{3x^2-12}$

$3(x^2-4) = 3(x+2)(x-2)$

Domain: $\mathbb{R}\ x \neq 2, -2$

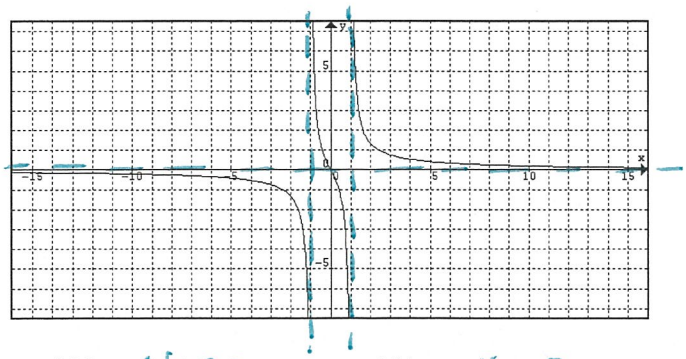
Vertical Asymptote: $x = 2,\ x = -2$

Horizontal Asymptote: $y = 2/3$

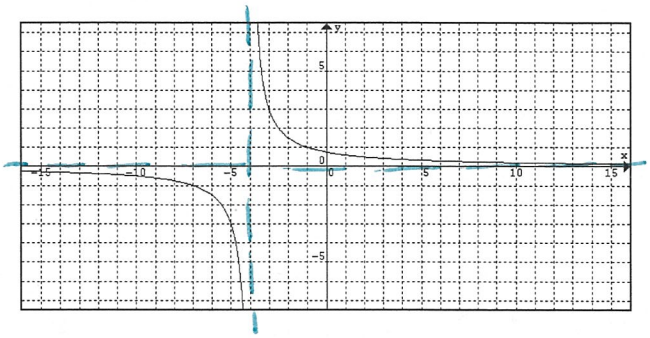
9. $f(x) = \dfrac{12x^4 + 10x - 3}{3x^4}$ Domain: $\mathbb{R}$ $x \neq 0$ $(-\infty, 0) \cup (0, \infty)$ Vertical Asymptote: $x = 0$ Horizontal Asymptote: $y = 4$	10. $f(x) = \dfrac{13x^4 + x^2}{6x + 3}$ Domain: $\mathbb{R}$, $x \neq -1/2$ $(-\infty, -1/2) \cup (-1/2, \infty)$ Vertical Asymptote: $x = -1/2$ Horizontal Asymptote: None
11. $f(x) = \dfrac{4}{2x^2 - 11x + 5}$ $(2x - 1)(x - 5)$ Domain: $\mathbb{R}$ $x \neq 1/2, 5$ Vertical Asymptote: $x = 1/2$, $x = 5$ Horizontal Asymptote: $y = 0$	12. $f(x) = \dfrac{11}{5x^3 - 45x}$ $5x(x^2 - 9)$ $5x(x+3)(x-3)$ Domain: $\mathbb{R}$, $x \neq 0, \pm 3$ Vertical Asymptote: $x = 0$, $x = 3$, $x = -3$ Horizontal Asymptote: $y = 0$

Find the vertical and horizontal asymptotes for the graphs of the given rational functions.

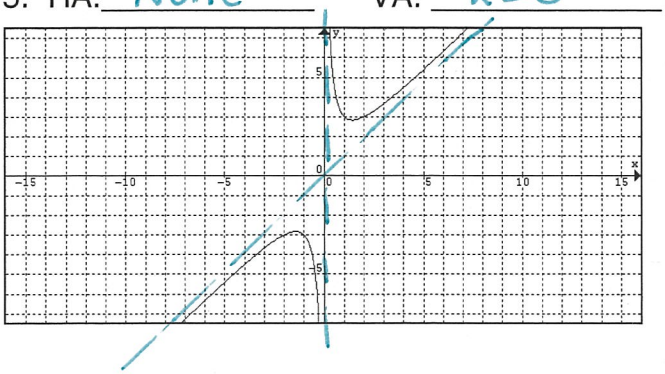
13. HA: $y = 0$ VA: $x = 1, x = -1$



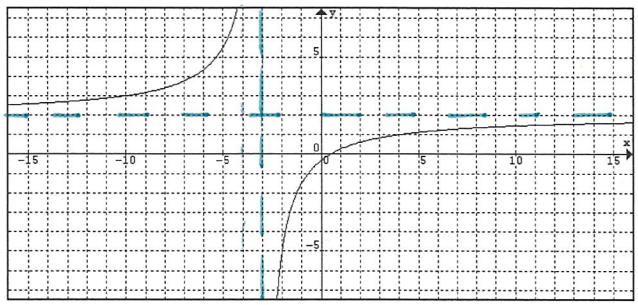
14. HA: $y = 0$ VA: $x = -4$



15. HA: None VA: $x = 0$

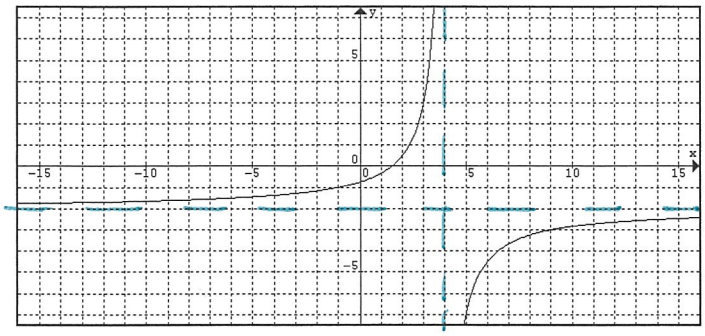


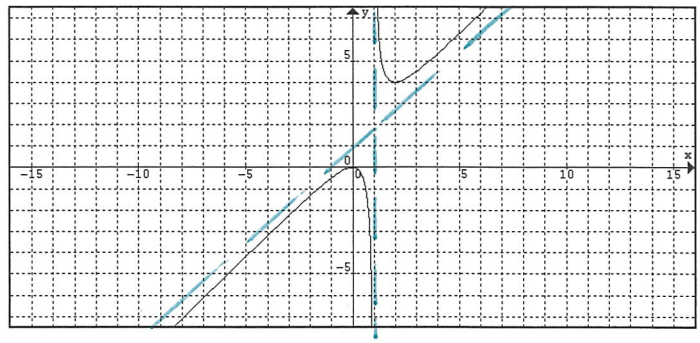
16. HA: $y = 2$ VA: $x = -3$ or -4



17. HA: $y=-2$ VA: $x=4$

18. HA: None VA: $x=1$





Analyze the given rational functions and then graph it.

19. $f(x) = \frac{2x^2-3x}{3x^2+x}$ $\frac{\cancel{x}(2x-3)}{\cancel{x}(3x+1)}$

a) holes: $(0,-3)$

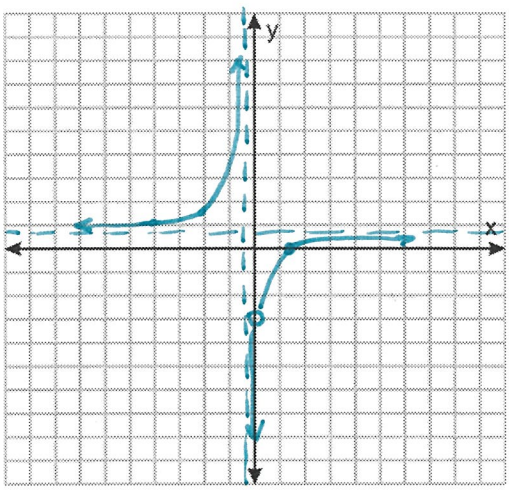
b) domain: $\mathbb{R}$ $x \neq -\frac{1}{3}, 0$

c) y-intercept: None (hole)

d) x-intercept(s): $(3/2, 0)$

e) vertical asymptotes: $x = -\frac{1}{3}$

f) horizontal asymptotes: $y = \frac{2}{3}$



20. $f(x) = \frac{1}{(x-2)^2}$

a) holes: None

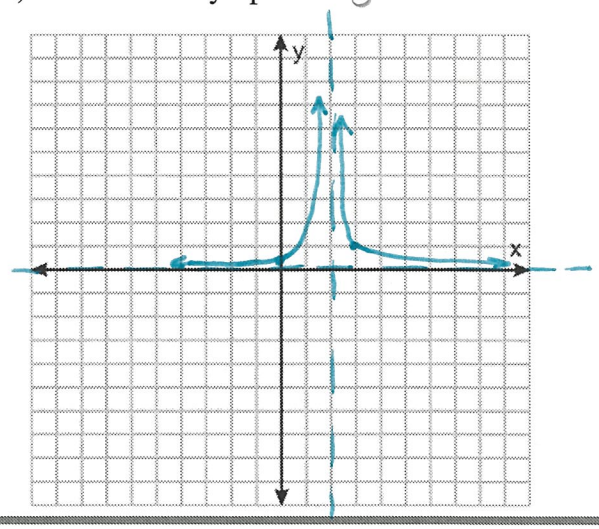
b) domain: $\mathbb{R}$ $x \neq 2$

c) y-intercept: $(0, 1/4)$

d) x-intercept(s): None

e) vertical asymptotes: $x=2$

f) horizontal asymptotes: $y=0$



21. $f(x) = \frac{x^3-4x}{x^2-x}$ $\frac{x(x^2-4)}{x(x-1)} = \frac{\cancel{x}(x+2)(x-2)}{\cancel{x}(x-1)}$

Holes: $(0,4)$

vertical asymptotes: $x=1$

horizontal asymptotes: None

22. $f(x)=\dfrac{2x^2-18}{x^2-16}$ $\dfrac{2(x^2-9)}{(x+4)(x-4)}=\dfrac{2(x+3)(x-3)}{(x+4)(x-4)}$

a) holes: None

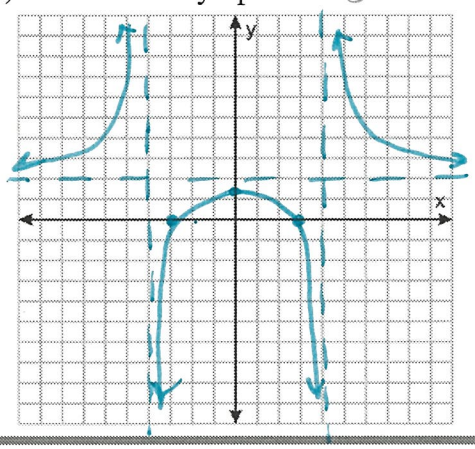
b) domain: $\mathbb{R}$ $x\neq 4,-4$

c) y-intercept: $(0, 9/8)$

d) x-intercept(s): $(-3,0)(3,0)$

e) vertical asymptotes: $x=4$, $x=-4$

f) horizontal asymptotes: $y=2$



23. $f(x)=\dfrac{5x^4}{x^4+1}$

a) holes: None

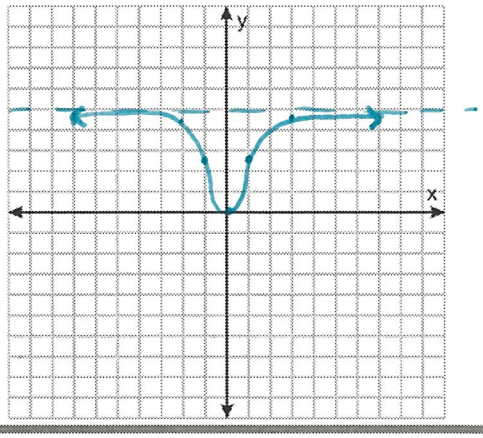
b) domain: $\mathbb{R}$

c) y-intercept: $(0,0)$

d) x-intercept(s): $(0,0)$

e) vertical asymptotes: None

f) horizontal asymptotes: $y=5$



24. $f(x)=\dfrac{x^2-2x}{x}$ $\dfrac{\cancel{x}(x-2)}{\cancel{x}}$

a) holes: $(0,-2)$

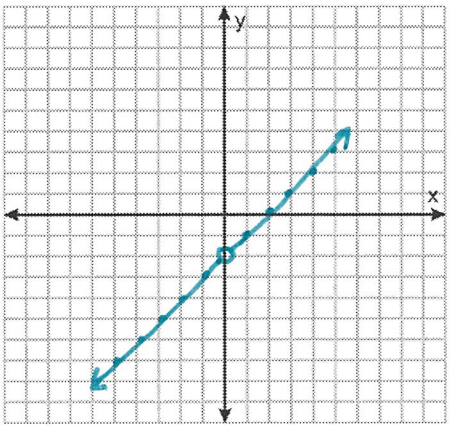
b) domain: $\mathbb{R}$, $x\neq 0$

c) y-intercept: None (hole)

d) x-intercept(s): $(2,0)$

e) vertical asymptotes: None

f) horizontal asymptotes: None



25. $f(x)=\dfrac{2x^2+5}{x^2-2x}$ $\dfrac{2x^2+5}{x(x-2)}$

a) holes: None

b) domain: $\mathbb{R}$ $x\neq 0,2$

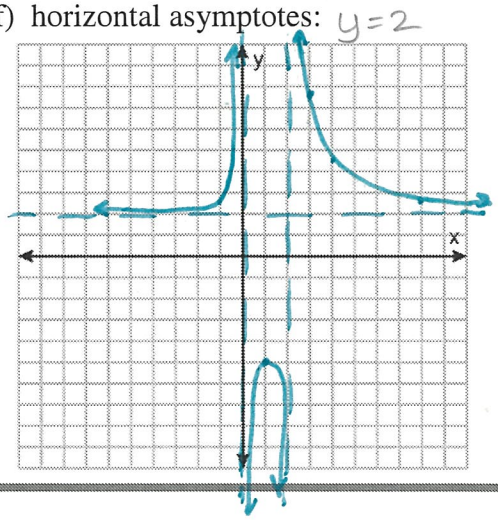
c) y-intercept: None (vertical asymptote)

d) x-intercept(s): None

$2x^2+5=0$
$2x^2=-5$
$x^2=\frac{-5}{2}$
$x=\pm\sqrt{\frac{-5}{2}}$

e) vertical asymptotes: $x=0$
$x=2$

f) horizontal asymptotes: $y=2$



26. $f(x) = \frac{3x-3}{x^2-1}$ $\frac{3(x-1)}{(x+1)(x-1)}$

a) holes: $(1, 3/2)$

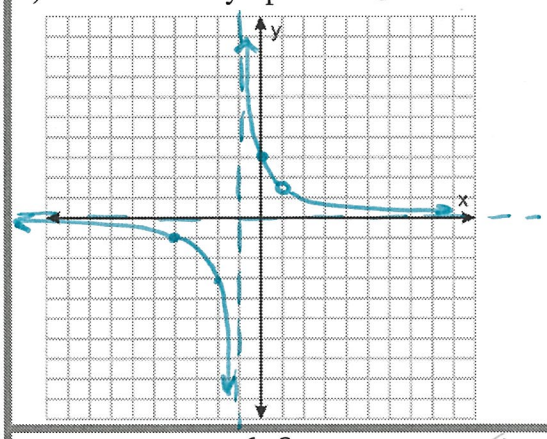
b) domain: $\mathbb{R}$ $x \neq 1, -1$

c) y-intercept: $(0, 3)$

d) x-intercept(s): None

e) vertical asymptotes: $x = -1$

f) horizontal asymptotes: $y = 0$



27. $f(x) = \frac{x^2}{x^2+1}$

a) holes: None

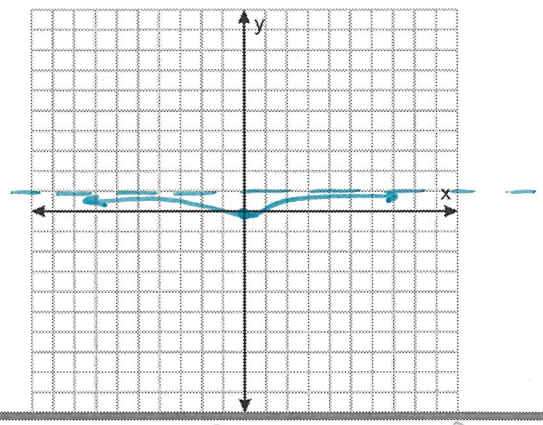
b) domain: $\mathbb{R}$

c) y-intercept: $(0, 0)$

d) x-intercept(s): $(0, 0)$

e) vertical asymptotes: None

f) horizontal asymptotes: $y = 1$



28. $f(x) = \frac{6-3x}{x^2-5x+6}$ $\frac{-3(x-2)}{(x-3)(x-2)}$

a) holes: $(2, 3)$

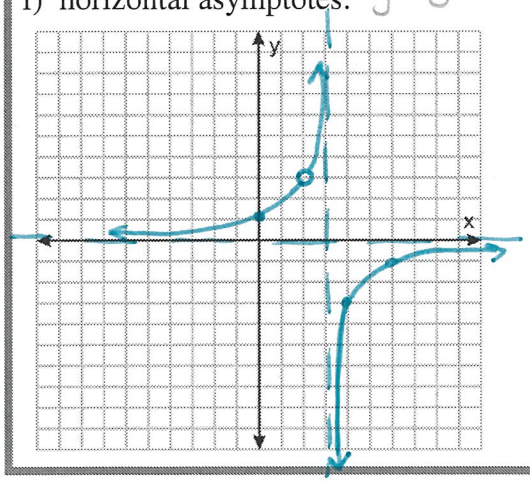
b) domain: $\mathbb{R}$ $x \neq 2, 3$

c) y-intercept: $(0, 1)$

d) x-intercept(s): None

e) vertical asymptotes: $x = 3$

f) horizontal asymptotes: $y = 0$



29. $f(x) = \frac{x^2-1}{x^4-1}$ $\frac{(x+1)(x-1)}{(x^2+1)(x^2-1)} =$

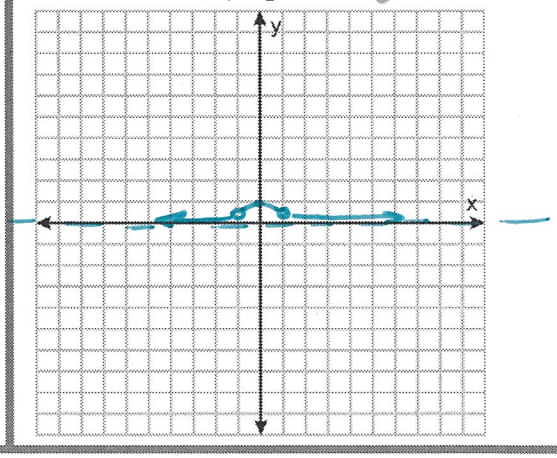
$= \frac{(x+1)(x-1)}{(x^2+1)(x+1)(x-1)}$

a) holes: $(1, 1/2)$ $(-1, 1/2)$

b) domain: $\mathbb{R}$ $x \neq 1, -1$

c) y-intercept: $(0, 1)$

d) x-intercept(s): None

e) vertical asymptotes: None

f) horizontal asymptotes: $y = 0$