23) $f(x) = \dfrac{5x^4}{x^4+1}$

$(x^2+i)(x^2-i)$

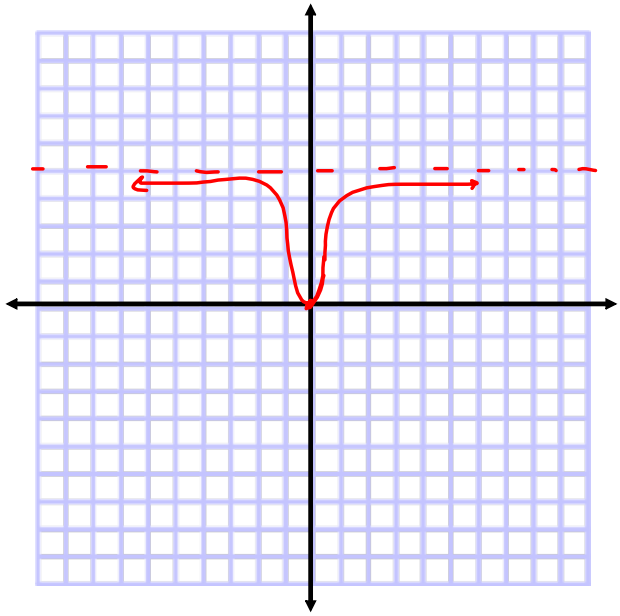
hole: None

y-int: (0,0)

x-int: (0,0)

VA: None

HA: $y=5$



Section 2.6 B

Rational Functions

Bell Work

Given the following function:

$$h(x) = \frac{2x - 4}{x^2 - 4} = \frac{2}{x+2}$$

$2\cancel{(x-2)}$

$(x+2)\cancel{(x-2)}$

Holes: $(2, \frac{1}{2})$

Vertical Asymptote: $x = 2$

Horizontal Asymptote: $y = 0$

Find the vertical and horizontal asymptotes without your calculator. ☺

1. $y = \dfrac{2x^2}{1x^2 - 9}$

$(x+3)(x-3)$

VA: $x = \pm 3$

HA: $y = 2$

2. $y = \dfrac{x^3 + 3x - 4}{2 - x^1}$

VA: $x = 2$

HA: None

3. $y = \dfrac{4}{x^2 + 2}$

VA: None

HA: $y = 0$

4. $y = \dfrac{1x^2}{3(x^2 - 1)}$

VA: $x = \pm 1$

HA: $y = \frac{1}{3}$

5. $g(x) = \dfrac{4x - 4}{2x^2 + x - 3}$

$\dfrac{4\cancel{(x-1)}}{(2x+3)\cancel{(x-1)}}$

VA: $x = -\frac{3}{2}$

HA: $y = 0$

6. $f(x) = \dfrac{4x - 2}{4x^2 - 1}$

$\dfrac{2\cancel{(2x-1)}}{(2x+1)\cancel{(2x-1)}}$

VA: $x = -\frac{1}{2}$

HA: $y = 0$

End behavior Asymptotes:

vertical asymptotes:

Set the factors of the denominator = 0, then solve for x

horizontal asymptotes: think of end behavior

Think about large values of x, what line do the answers approach?

Compare the degree & leading coefficient of the numerator and denominator:

$$f(x) = \frac{nx^a\ldots}{mx^b\ldots}$$

horizontal

a) a < b, y = 0

b) a = b, y = n/m (ratio of leading coefficients)

c) a > b, y = higher degree polynomial

Focusing on case 3 today where the degree of the numerator is bigger than the degree of the denominator.

**They are not classified as horizontal or vertical asymptotes.

**There will not be a horizontal asymptote in this case.

Naming higher degree end behavior asymptotes:

Find the difference in the degrees of the numerator and denominator.

Function	Name the higher degree end behavior asympotote.
$f(x)=\dfrac{x^{4}+2x^{2}-1}{x^{3}+1}$	difference degree 1 $y=x^{1}$ Slant/linear
$f(x)=\dfrac{x^{4}-2x+7}{x^{2}-1}$	difference degree 2 $y=x^{2}$ parabolic parabola quadratic
$f(x)=\dfrac{x^{4}-1}{x^{1}-1}$	difference degree 3 $y=x^{3}$ cubic degree 3

Given the following function find the end behavior asymptote and name it.

$y = x^2 - 2x + 1$

$$f(x) = \frac{x^3 - 3x^2 + 3x + 1}{x - 1}$$

$x - 1 \overline{\big) x^3 - 3x^2 + 3x + 1}$

1	1	-3	3	1
	↓	1	-2	1
	1	-2	1	~~0~~

remainder

$y = x^2 - 2x + 1$

Given the following function find the end behavior asymptote and name it. $\boxed{\text{Slant}}$

$y = 2x - 4$ Slant

$$f(x) = \frac{2x^2 + 2x - 3}{x^1 + 3}$$

-3	2	2	-3
	↓	-6	12
	2	-4	~~9~~

Given the following function find the end behavior asymptote and name it. cubic $y = x^3 - x$

$$f(x) = \frac{x^5 + 1}{x^2 + 1}$$

$$\begin{array}{r|l} & x^3 \quad -x \\ \hline x^2 + 0x + 1 & x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 1 \\ & -x^5 + 0x^4 + x^3 \\ \hline & 0 \quad 0 \quad -x^3 + 0x^2 + 0x \\ & \quad\quad +x^3 + 0x^2 + x \\ \hline & \quad\quad\quad x + 1 \end{array}$$

Draw the graph of the rational function and show all asymptotes.

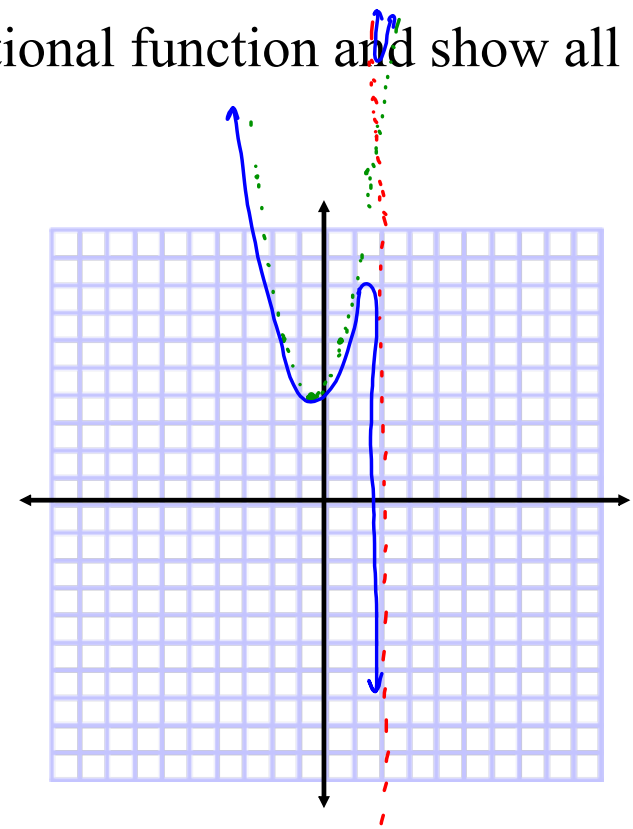
$$g(x) = \frac{2x^3 - 2x^2 - x - 5}{x - 2}$$

VA: $x = 2$

HA: None

$$\begin{array}{r|rrrr} 2 & 2 & -2 & -1 & -5 \\ & \downarrow & 4 & 4 & 6 \\ \hline & 2 & 2 & 3 & 1 \end{array}$$

$y = 2x^2 + 2x + 3$



$(x+1)(x^2-x+1)$

$$f(x)=\frac{x^3+1}{x-1}$$

Hole: None

Vertical asymptote: $x=1$

Horizontal asymptote: None

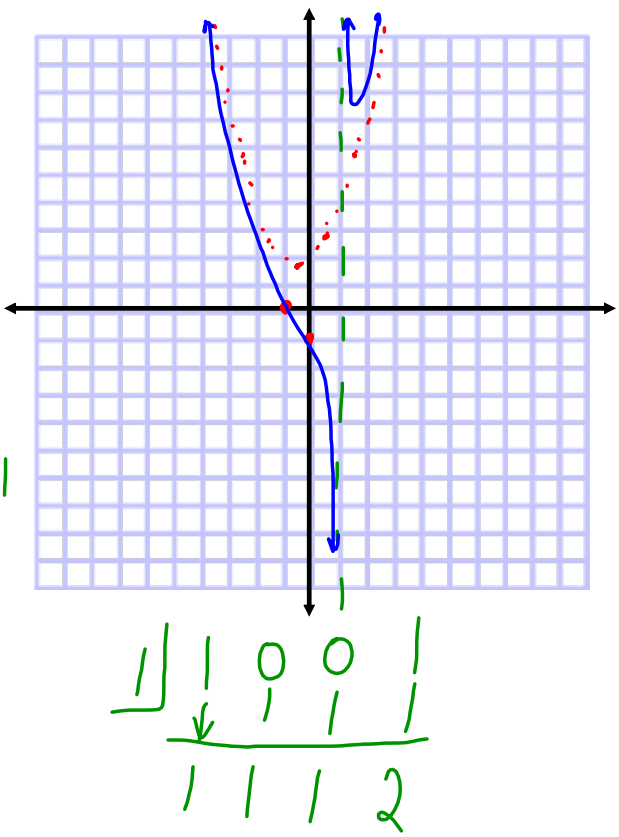
End behavior asymptote: $y=x^2+x+1$

x – Intercept: $(-1,0)$

y – Intercept: $(0,-1)$

Domain: $\mathbb{R}, x\neq 1$

Range: $\mathbb{R}$



$$f(x)=\frac{x^2-x-2}{x-3}$$

Hole:

Vertical asymptote:

Horizontal asymptote:

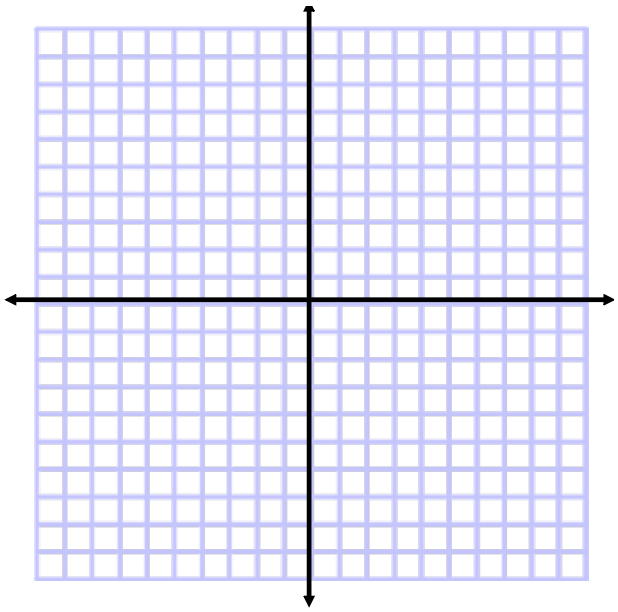
End behavior asymptote:

x – Intercept:

y – Intercept:

Domain:

Range:



Worksheet 2.6B