Precalculus
Section 2.6 B

Name Key
Period_____ Date_____

Graph the rational function and answer the given questions. Show ALL asymptotes.

1. $g(x) = \frac{x-2}{x^2-3x-4}$ $\frac{(x-2)}{(x-4)(x+1)}$

Hole: None

Vertical asymptote: $x=4,\ x=-1$

Horizontal asymptote: $y=0$

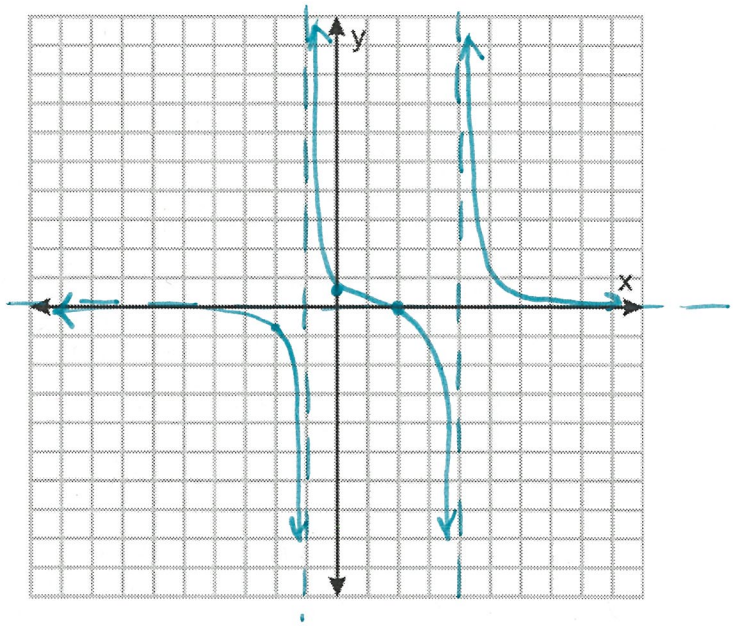
End behavior asymptote: None

x – Intercept: $(2,0)$

y – Intercept: $(0, 1/2)$

Domain: $\mathbb{R},\ x \neq 4, -1$

Range: $\mathbb{R},\ y \neq 0$



2. $h(x) = \frac{3}{x^3-x}$ $\frac{3}{x(x^2-1)} = \frac{3}{x(x+1)(x-1)}$

Hole: None

Vertical asymptote: $x=0,\ x=-1,\ x=1$

Horizontal asymptote: $y=0$

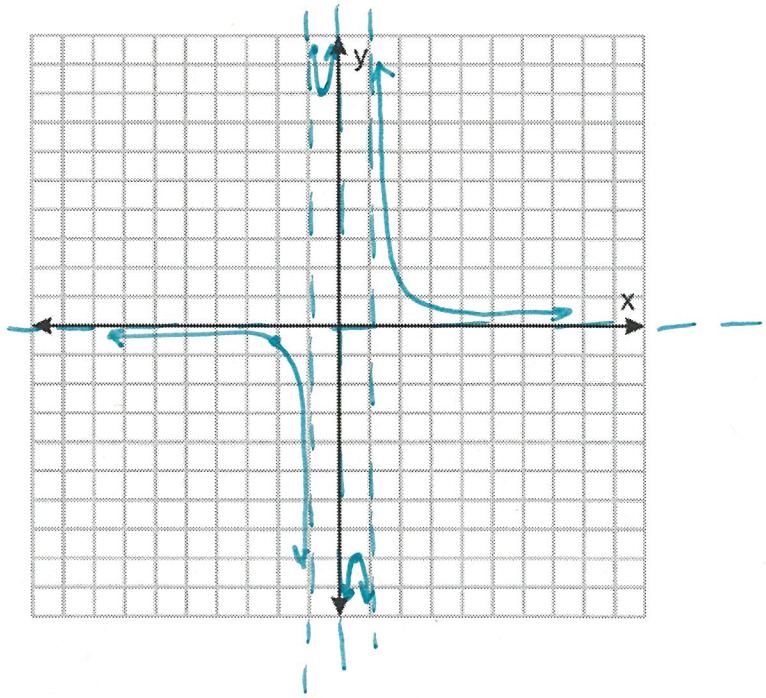
End behavior asymptote: None

x – Intercept: None

y – Intercept: None

Domain: $\mathbb{R}$ $x \neq 0, -1, 1$

Range: $\mathbb{R}$ $y \neq 0$



3. $f(x) = \frac{x^3-9x}{3x^2-6x-9}$ $\frac{x(x^2-9)}{3(x^2-2x-3)} = \frac{x(x+3)\cancel{(x-3)}}{3\cancel{(x-3)}(x+1)}$

3(6)

3(4)

$$\begin{array}{r} \frac{1}{3}x + \frac{2}{3} \\ 3x^2-6x-9 \overline{\big) x^3+0x^2-9x+0} \\ -x^3+2x^2+3x \\ \hline 2x^2-6x \\ 2x^2 \end{array}$$

Hole: $(3, 3/2)$

Vertical asymptote: $x=-1$

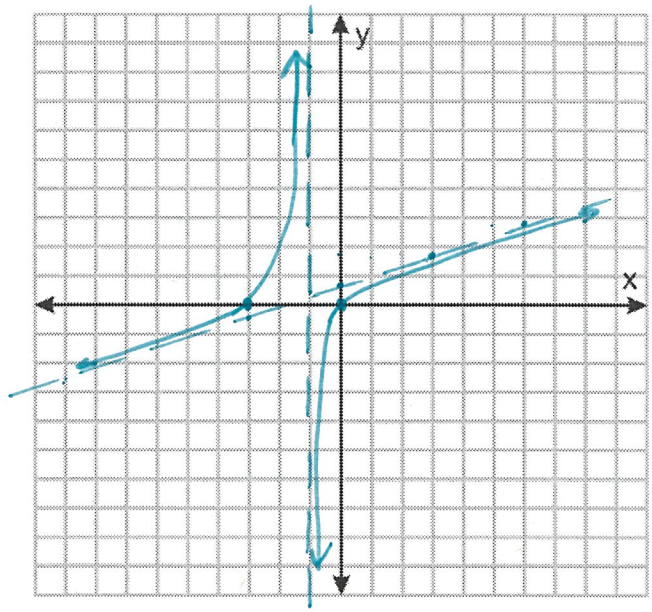
Horizontal asymptote: None

End behavior asymptote: $y = \frac{1}{3}x + \frac{2}{3}$ (slant)

x – Intercept: $(0,0)$ $(-3,0)$

y – Intercept: $(0,0)$

Domain: $\mathbb{R}$, $x \neq -1, 3$



4. $f(x) = \frac{2x^2-4x+1}{x^2+2x}$ $\frac{2x^2-4x+1}{x(x+2)}$

$2x^2-4x+1=0$

$x = \frac{4 \pm \sqrt{16-4(2)(1)}}{2(2)} = \frac{4 \pm \sqrt{8}}{4} = \frac{4 \pm 2\sqrt{2}}{4}$

Hole: None

Vertical asymptote: $x = 0, -2$

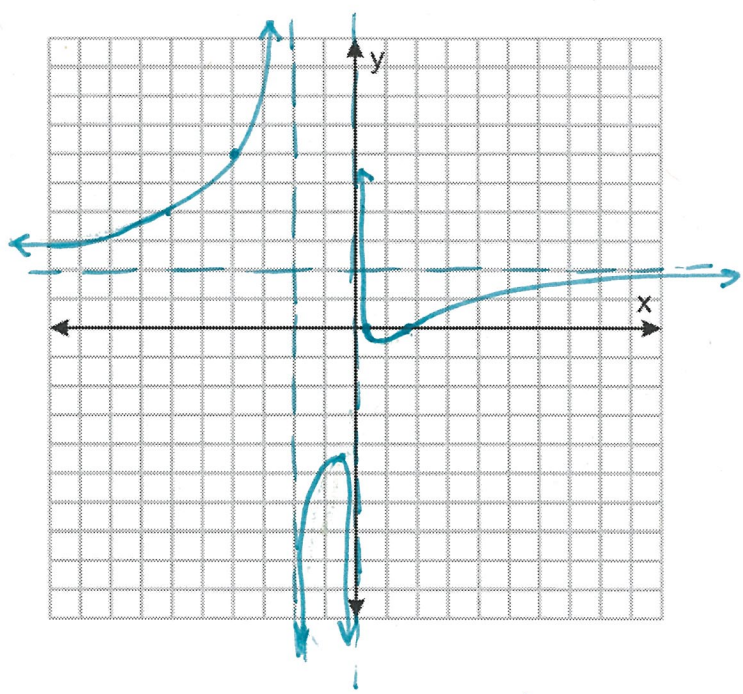
Horizontal asymptote: $y = 2$

End behavior asymptote: None

x – Intercept: $(1 \pm \frac{1}{2}\sqrt{2}, 0) \approx (1.7, 0)$ $(.29, 0)$

y – Intercept: None

Domain: $\mathbb{R}$, $x \neq 0, -2$



5. $f(x) = \frac{2x+2}{2x^2-x-3}$ $\frac{2(x+1)}{(2x-3)(x+1)}$

Hole: $(-1, -2/5)$

Vertical asymptote: $x = 3/2$

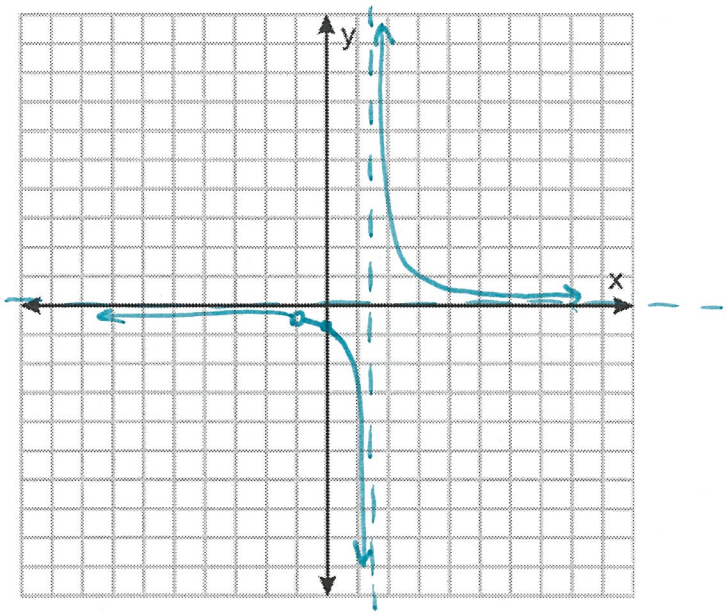
Horizontal asymptote: $y = 0$

End behavior asymptote: None

x – Intercept: None

y – Intercept: $(0, -2/3)$

Domain: $\mathbb{R}$, $x \neq 3/2, -1$



6. $h(x) = \frac{x-1}{x^2-x-6}$ $\frac{(x-1)}{(x-3)(x+2)}$

Hole: None

Vertical asymptote: $x = 3, -2$

Horizontal asymptote: $y = 0$

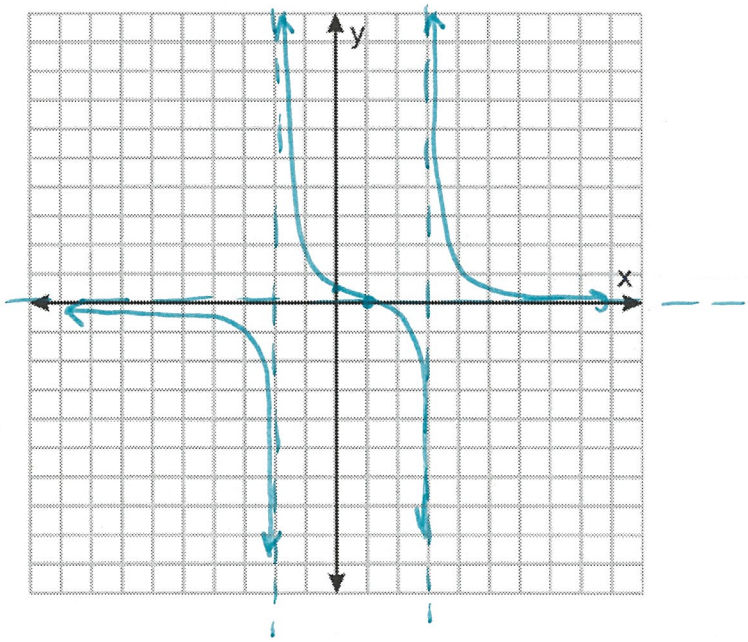
End behavior asymptote: None

x – Intercept: $(1, 0)$

y – Intercept: $(0, 1/6)$

Domain: $\mathbb{R}$ $x \neq 3, -2$

Range: $\mathbb{R}$ $y \neq 0$



7. $f(x) = \frac{x^2-2x+3}{x-5}$

Hole: None

Vertical asymptote: $x=5$

Horizontal asymptote: None

End behavior asymptote: $y=x+3$ (slant)

x – Intercept: None

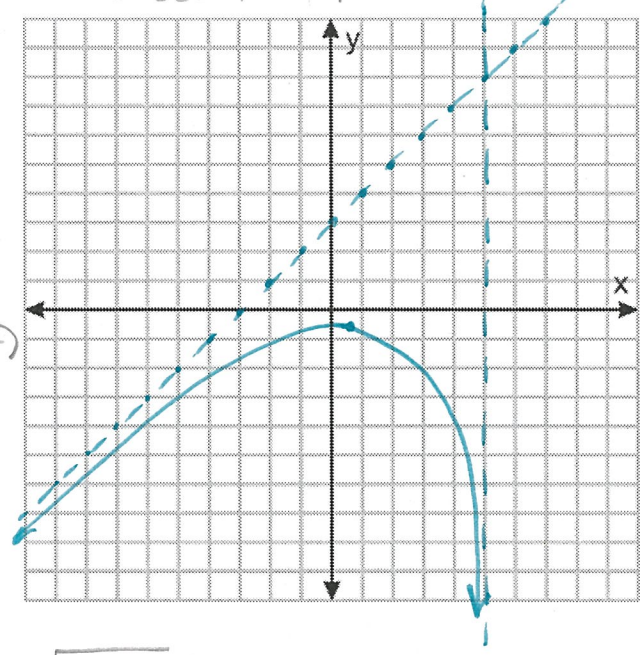
y – Intercept: $(0, -3/5)$

Domain: $\mathbb{R}, x \neq 5$

$$\begin{array}{r} x+3 \\ x-5 \overline{)x^2-2x+3} \\ -x^2+5x \\ \hline 3x+3 \\ -3x+15 \\ \hline 18 \end{array}$$

$$x = \frac{2 \pm \sqrt{4-4(1)(3)}}{2(1)}$$

change window to see this part



8. $g(x) = \frac{x^3-8}{x+2}$ $\frac{(x-2)(x^2+2x+4)}{(x+2)}$

Hole: None

Vertical asymptote: $x=-2$

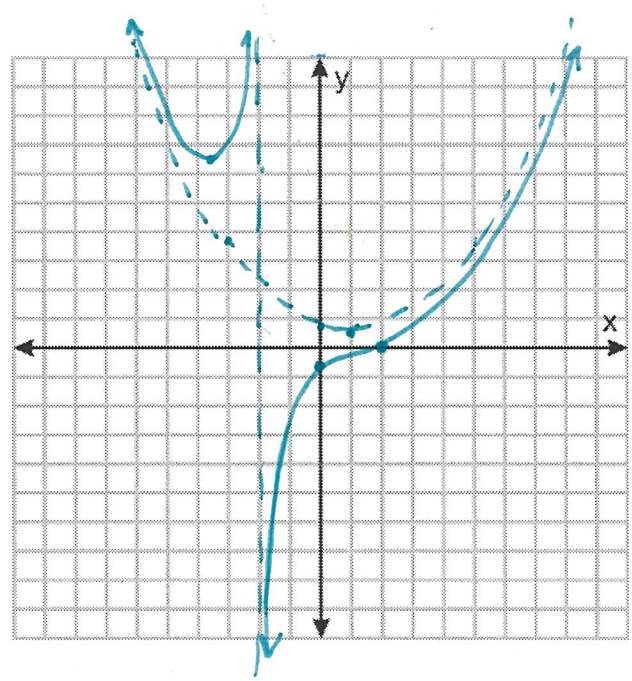
Horizontal asymptote: None

End behavior asymptote: $y=x^2-2x+4$ (parabolic)

x – Intercept: $(2,0)$

y – Intercept: $(0,-4)$

$$\begin{array}{r} x^2-2x+4 \\ x+2 \overline{)x^3+0x^2+0x-8} \\ -x^3+2x^2 \\ \hline -2x^2+0x \\ +2x^2+4x \\ \hline 4x-8 \end{array}$$



rescale y-count by 5

9. $f(x) = \frac{x^4-2x+1}{x+2}$

Hole: None

Vertical asymptote: $x=-2$

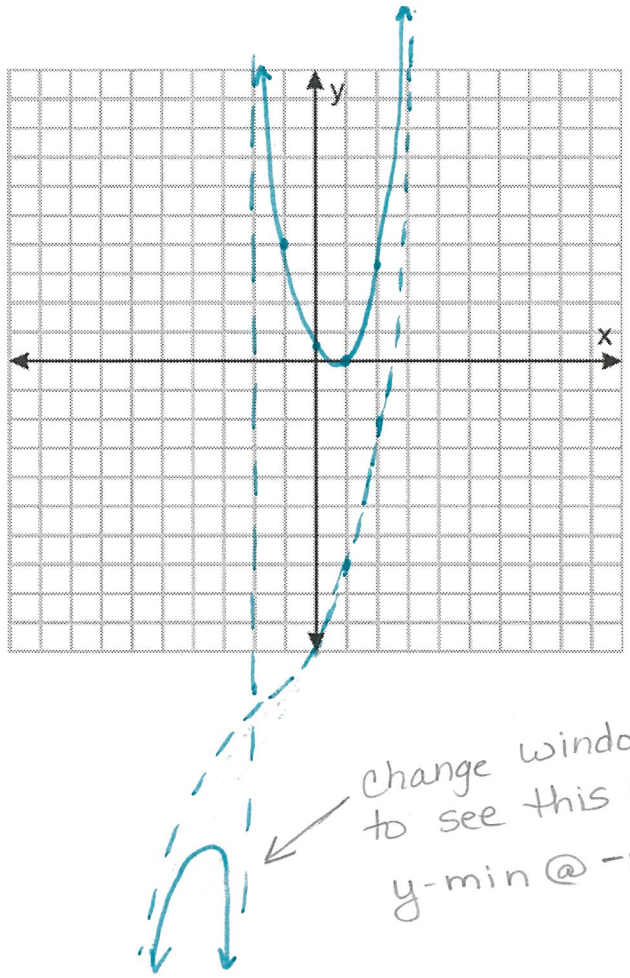
Horizontal asymptote: None

End behavior asymptote: $y = x^3-2x^2+4x-10$ (cubic)

x – Intercept:

y – Intercept: $(0, 1/2)$

Domain: $\mathbb{R}$ $x \neq -2$



-2	1	0	0	-2	1
	↓	-2	4	-8	20
	1	-2	4	-10	21

$y = x^3-2x^2+4x-10$

10. $f(x) = \frac{x^2+3x+2}{x^2-2x-3}$ $\frac{(x+2)\cancel{(x+1)}}{(x-3)\cancel{(x+1)}}$

Hole: $(-1, -1/4)$

Vertical asymptote: $x=3$

Horizontal asymptote: $y=1$

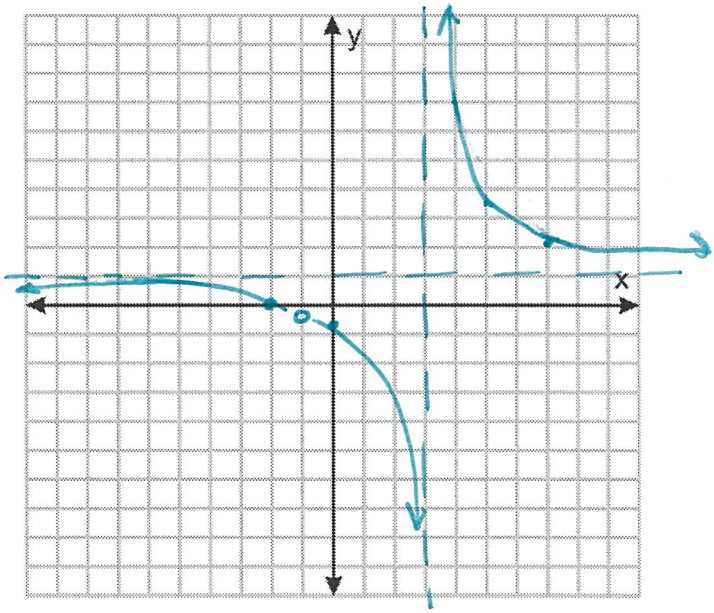
End behavior asymptote: None

x – Intercept: $(-2,0)$

y – Intercept: $(0,-2/3)$

Domain: $\mathbb{R}$ $x \neq 3, -1$

Range: $\mathbb{R}$ $y \neq 1$



11. $h(x) = \frac{x^3-27}{x-3}$ $\frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}}$

Hole: (3,27)

Vertical asymptote: None

Horizontal asymptote: None

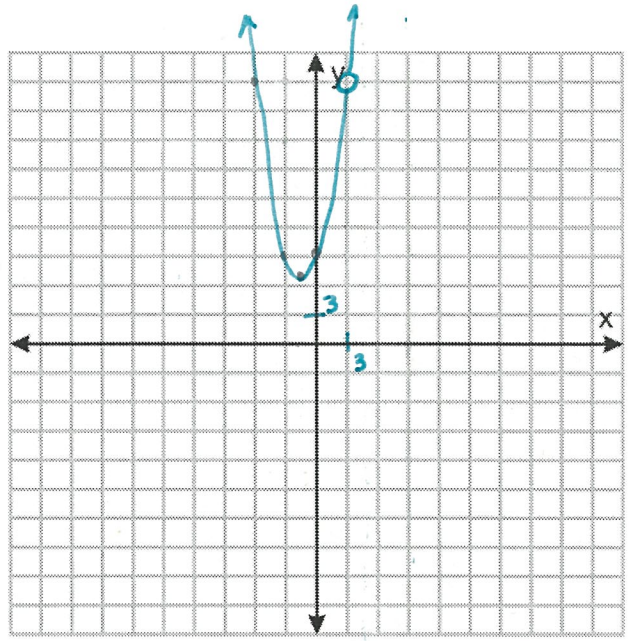
End behavior asymptote: None

x – Intercept: None

y – Intercept: (0,9)

$$\begin{array}{r|rrrr} 3 & 1 & 0 & 0 & -27 \\ & \downarrow & 3 & 9 & 27 \\ \hline & 1 & 3 & 9 & 0 \end{array}$$

x^2+3x+9



change scale
x & y count by 3

12. $h(x) = \frac{x^2+2x-3}{x+1}$ $\frac{(x+3)(x-1)}{(x+1)}$

Hole: None

Vertical asymptote: $x = -1$

Horizontal asymptote: None

End behavior asymptote: $y = x+1$ (slant)

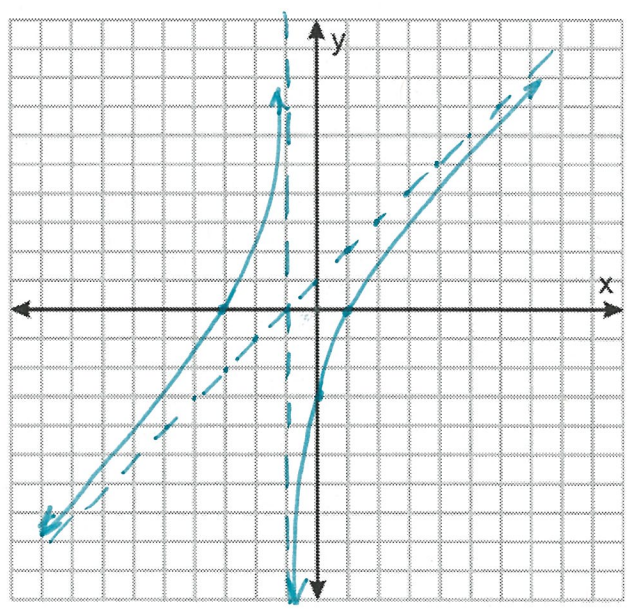
x – Intercept: (-3,0) (1,0)

y – Intercept: (0,-3)

Domain: $\mathbb{R}$ $x \neq -1$

Range: $\mathbb{R}$

$$\begin{array}{r|rrr} -1 & 1 & 2 & -3 \\ & \downarrow & -1 & -1 \\ \hline & 1 & 1 & -4 \end{array}$$



13. $f(x) = \frac{x^2+x-2}{x^2-4}$ $\frac{(x+2)(x-1)}{(x+2)(x-2)}$

Hole: $(-2, \frac{1}{2})$

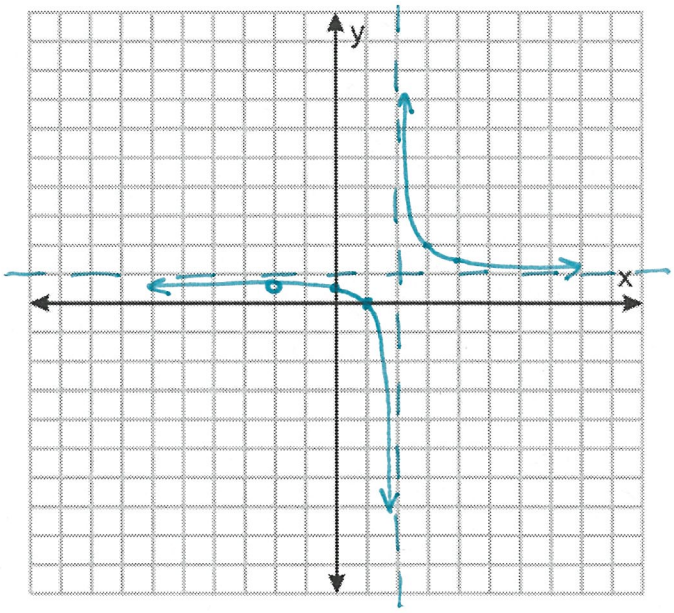
Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 1$

End behavior asymptote: None

x – Intercept: $(1, 0)$

y – Intercept: $(0, \frac{1}{2})$

Domain: $\mathbb{R}$ $x \neq 2$

Range: $\mathbb{R}$ $y \neq 1$