26) $\log_3(x) - \log_3(x+3) = \log_3 24$

$\log_3 \frac{x}{x+3} = \log_3 24$

$\frac{x}{x+3} = \frac{24}{1}$

$x = 24(x+3)$

$x = 24x + 72$

$-23x = 72$

$x = \frac{72}{-23}$

No Solution

3.5 Exponential and Logarithmic Models

The five most common types of mathematical models involving exponential functions and logarithmic functiosn are as follows:

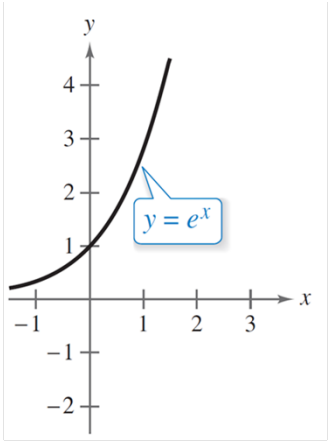
1. Exponential growth model; $y = ae^{bx},\ b > 0$

2. Exponential decay model; $y = ae^{-bx},\ b > 0$

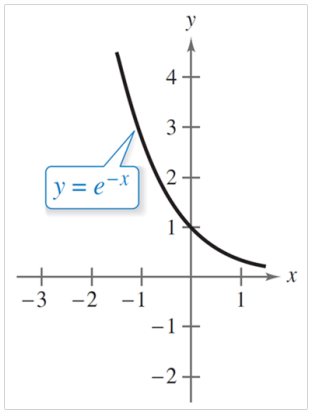
3. Gaussian Model; $y = ae^{-(x-b)^2/c}$

4. Logistic growth model; $y = \frac{a}{1 + be^{-rx}}$

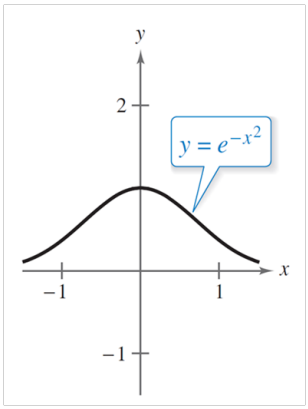
5. Logarithmic models; $y = a + b \ln x,\ y = a + b \log x$



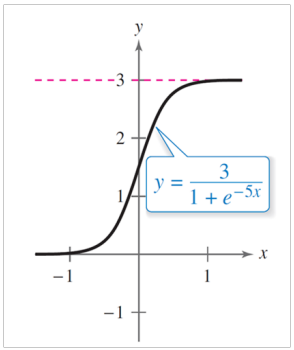
Exponential growth model



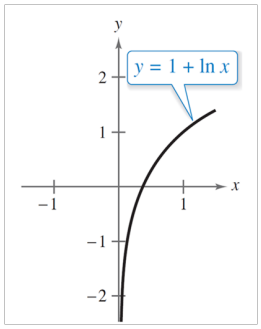
Exponential decay model



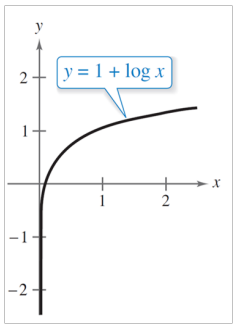
Gaussian model



Logistic growth model



Natural logarithmic model



Common logarithmic model

Interest Compounded Continuously

$A = Pe^{rt}$

P = principal

r = fixed annual interest rate

t = time in years

Interest compounded k times per year

$A = P(1 + \frac{r}{k})^{kt}$

k = number of times compounded

kt = number of compounding periods

k = number of times compounded

t = time in years

Find the missing values assuming continuously compounded interest.

	Initial Investment	Annual % Rate	Time to Double	Amount after 10 yrs
1)	$750	10.5% .105	6.601 yrs	$A=750e^{(.105)(10)} = \$2143.24$
2)	$500	11%	6.3 yrs	$1505.00
3)	$1119.79	5.8%	12 yr	$2000.00

1) $A = Pe^{rt}$ $1500 = 750e^{.105t}$ $2 = e^{.105t}$ $\ln_e 2 = .105t$ $\frac{\ln 2}{.105} = t$

2) $A = Pe^{rt}$ $1505.00 = 500e^{r(10)}$ $3.01 = e^{10r}$ $\ln_e 3.01 = 10r$ $r = .110$ $2 = e^{.11t}$ $\ln 2 = .11t$

3) $2 = e^{r(12)}$ $\ln 2 = 12r$ $r = .0577$ $A = Pe^{rt}$ $2000 = Pe^{(.058)(10)}$ $2000 = 1.786P$ $P = \$1119.79$

Determine the principal P that must be invested at rate *r*, compounded monthly, so that $500,000 will be available for retirement in *t* years.

$r = 4\frac{1}{2}\%$ and $t = 10$

$$A = P(1 + \frac{r}{k})^{kt}$$

$500{,}000 = P(1 + \frac{.045}{12})^{12(10)}$

$500{,}000 = 1.566P$

$P = \$319{,}082.51$

Determine the time necessary for P dollars to double when it is invested at interest rate r compounded

a) annually, b) monthly, c) daily, d) continuously $A = Pe^{rt}$ $2 = e^{rt}$

8.665yrs 8.66yrs

$r = 8\%$

$$A = P(1 + \frac{r}{k})^{kt}$$

a) $2 = (1 + \frac{.08}{1})^{(1)(t)}$

$2 = (1.08)^t$

$\log_{1.08} 2 = t$

$\frac{\log 2}{\log 1.08} = t$

$t = 9$ yrs.

b) $2 = (1 + \frac{.08}{12})^{12t}$ $2 = (1 + \frac{r}{k})^{kt}$

$2 = (1.0066)^{12t}$

$2 = 12t$

$\log_{1.0066}$

$t = 8.69$

Find the missing value for the radioactive isotope.

Isotope	Half-life (years)	Initial Quantity	Amount After 1000 yrs
^{14}C	5715	6.5 g	5.76g
^{239}PU	24,100	.412g	0.4 g

$P(t) = 6.5(\frac{1}{2})^{\frac{1000}{5715}}$

Final = 5.76g

$.4 = P(\frac{1}{2})^{\frac{1000}{24,100}}$

Pattern for doubling populations, tripling, etc.

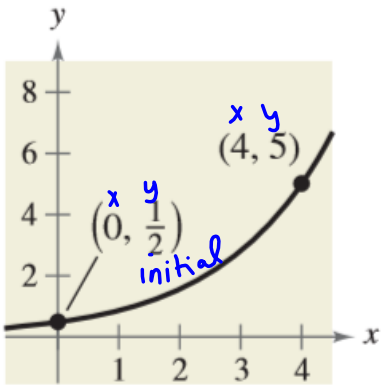
$y = (\text{initial amount})(\text{growth factor})^{x/\text{time}}$

Half-life formula

$P(t) = P_o(1/2)^{t/h}$

$P_o \Rightarrow$ initial value
$t \Rightarrow$ time
$h \Rightarrow$ half-life

Find the exponential model $y = ae^{bx}$ that fits the points shown in the graph or table.



$y = ae^{bx}$

$y = \frac{1}{2}e^{bx}$

$5 = \frac{1}{2}e^{4b}$

$10 = e^{4b}$

$\ln 10 = 4b$

$b = \frac{\ln 10}{4}$

$y = \frac{1}{2}e^{\frac{\ln 10}{4}x}$

Gaussian Models

As mentioned at the beginning of this section, Gaussian models are of the form

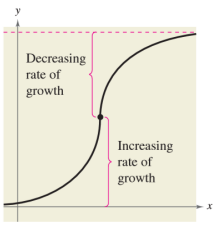
$$y = ae^{-(x-b)^2/c}.$$

This type of model is commonly used in probability and statistics to represent populations that are **normally distributed.** The graph of a Gaussian model is called a **bell-shaped curve.** Try graphing the normal distribution with a graphing utility. Can you see why it is called a bell-shaped curve?

For *standard* normal distributions, the model takes the form

$$y = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}.$$

The **average value** of a population can be found from the bell-shaped curve by observing where the maximum y-value of the function occurs. The x-value corresponding to the maximum y-value of the function represents the average value of the independent variable—in this case, x.



Logistic Growth Models

Some populations initially have rapid growth, followed by a declining rate of growth, as indicated by the graph in Figure 3.40. One model for describing this type of growth pattern is the **logistic curve** given by the function

$$y = \frac{a}{1 + be^{-rx}}$$

where y is the population size and x is the time. An example is a bacteria culture that is initially allowed to grow under ideal conditions, and then under less favorable conditions that inhibit growth. A logistic growth curve is also called a **sigmoidal curve.**

On a college campus of 5000 students, one student returns from vacation with a contagious and long-lasting flu virus. The spread of the virus is modeled by

$$y = \frac{5000}{1 + 4999e^{-0.8t}}, \quad t \geq 0$$

where y is the total number of students infected after t days. The college will cancel classes when 40% or more of the students are infected.

a. How many students are infected after 5 days?

b. After how many days will the college cancel classes?

$y = \frac{5000}{1+4999e^{-.8(5)}}$

$y = 54$ students

$\frac{2000}{1} = \frac{5000}{1+4999e^{-.8t}}$

$1+4999e^{-.8t} = \frac{5000}{2000}$

$4999e^{-.8t} = 1.5$

$e^{-.8t} = \frac{1.5}{4999}$

$\ln\left(\frac{1.5}{4999}\right) = -.8t$

$t = 10.13$ days

Section 3.5 Pg. 243-247: #7-11odd, 21, 23, 25-28, 41-43, 66