Precalculus Name Key

6.1 & 6.2 Law of sines and cosines applications Period_____Date_____

1. Observers 2.32 mi apart see a hot-air balloon directly between them but at the angles of elevation shown in the figure. Find the altitude of the balloon.

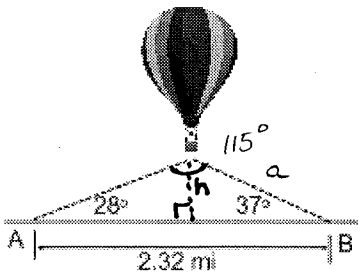
$\frac{a}{\sin 28^\circ} = \frac{2.32}{\sin 115^\circ}$

$a = 1.2$ mi

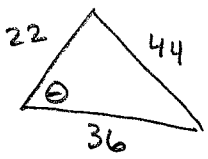
$\sin 37^\circ = \frac{h}{1.2}$

$h = 1.2 \sin 37^\circ$

$h = .72$ miles



2. A triangular field has sides of lengths 22, 36, 44 yd. Find the largest angle.



$44^2 = 22^2 + 36^2 - 2(22)(36)\cos\theta$

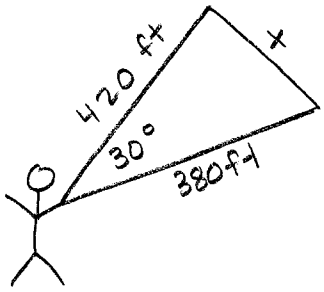
$1936 = 1780 - 1584\cos\theta$

$156 = -1584\cos\theta$

$\frac{156}{-1584} = \cos\theta$

$\theta = \cos^{-1}(-156/1584)$ $\theta = 95.7^\circ$

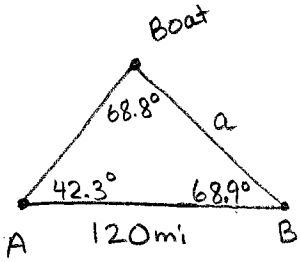
3. A boy is flying two kites at the same time. He has 380 ft of line out to one kite and 420 ft to the other. He estimates the angle between the two lines to be 30°. Approximate the distance between the kites.



$x^2 = 420^2 + 380^2 - 2(420)(380)\cos 30^\circ$

$x \approx 210.6$ ft

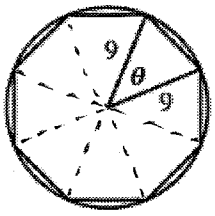
4. A boat is cruising the ocean off a straight shore line. Points A and B are 120 mi apart on the shore. It is found that angles from the points to the boat are 42.3° and 68.9° respectively. Find the shortest distance from the boat to the shore.



$\frac{a}{\sin 42.3^\circ} = \frac{120}{\sin 68.8^\circ}$

$a = 86.6$ miles

5. Find the area of a regular octagon (8 equal sides, 8 equal angles) inscribed inside a circle of radius 9 inches.



$\theta = 45^\circ$

$\frac{360^\circ}{8} = 45^\circ$

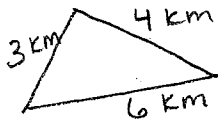
$A = \frac{1}{2}(9)(9)\sin 45^\circ$

$A = 28.63 \text{ in}^2$

↑ one triangle (multiply by 8)

$A = 229.1 \text{ in}^2$

6. Because deer require food, water, cover for protection from weather and predators and living space for healthy survival, there are natural limits to the number of deer that a given plot of land can support. Deer population in national parks average 14 animals per square kilometer. If a triangular region with side of 3 kilometers, 4 kilometers, and 6 kilometers has a population of 50 deer, how close is the population on this land to the average national park population?



$S = \left(\frac{3+4+6}{2}\right)$

$S = 6.5$

$A = \sqrt{6.5(6.5-3)(6.5-4)(6.5-6)}$

$A = \sqrt{28.4375}$

$A = 5.33 \text{ km}^2$

population $\frac{50}{5.33} = 9.38 \text{ Deer/km}^2$

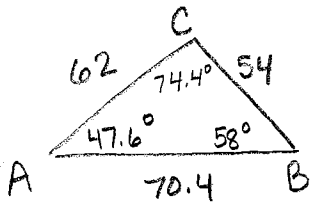
less than national average of 14

7. ◆ **MULTIPLE REPRESENTATIONS** You are fertilizing a triangular garden. One side of the garden is 62 feet long and another side is 54 feet long. The angle opposite the 62 foot side is 58°.

   a. **Drawing a Diagram** Draw a diagram to represent this situation.

   b. **Solving a Triangle** Use the law of sines to solve the triangle you drew in part (a).

   c. **Applying a Formula** One bag of fertilizer covers an area of 200 square feet. How many bags of fertilizer will you need to cover the entire garden?



$\frac{\sin A}{54} = \frac{\sin 58^\circ}{62}$

$\angle A = 47.6^\circ$

$\angle C = 74.4^\circ$

$\frac{c}{\sin 74.4^\circ} = \frac{62}{\sin 58^\circ}$

$C = 70.4$

Area triangle

$A = \frac{1}{2}(62)(54)\sin 74.4^\circ$

$A = 1612.33 \text{ ft}^2$

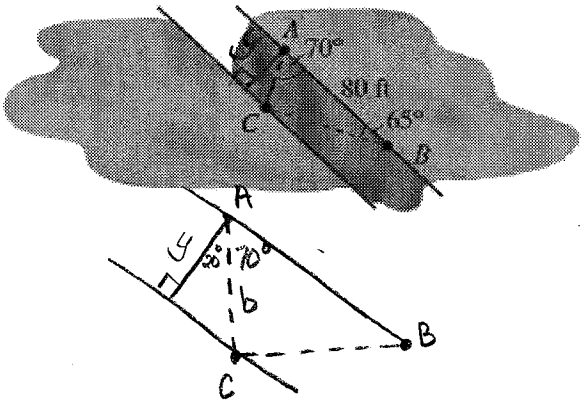
$\frac{1612.33}{200} = 8.06$

9 bags

8. Two markers A and B on the same side of a canyon rim are 80 ft apart, as shown in the figure. A hiker is located across the rim at point C. A surveyor determines that $\angle BAC = 70°$ and $\angle ABC = 65°$.

a) What is the distance between the hiker and point A? 102.5 ft

b) What is the distance between the two canyon rims? (Assume they are parallel) 96.4 ft



$$\frac{b}{\sin 65°} = \frac{80}{\sin 45°}$$

$$b = \frac{80 \sin 65°}{\sin 45°}$$

$$b = 102.5 \text{ ft}$$

$$\cos 20° = \frac{y}{102.54}$$

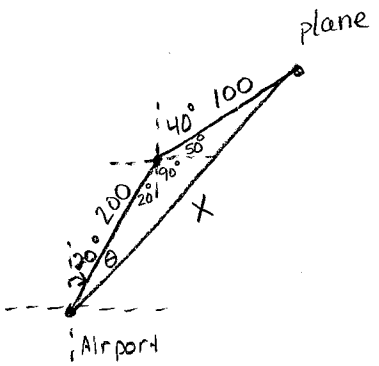
$$y = 102.54 \cos 20°$$

$$y = 96.4$$

9. A pilot sets out from an airport and heads in the direction of N20°E, flying at 200 mph. After one hour, he makes a course correction and heads in the direction of N40°E. Half an hour after that, engine trouble forces him to make an emergency landing.

a) Find the distance between the airport and his final landing point. 295.95 miles

b) Find the bearing from the airport to his final landing.



$$x^2 = 200^2 + 100^2 - 2(200)(100)\cos 160°$$

$$x = 295.95 \text{ miles}$$

$$\frac{\sin\theta}{100} = \frac{\sin 160°}{295.95}$$

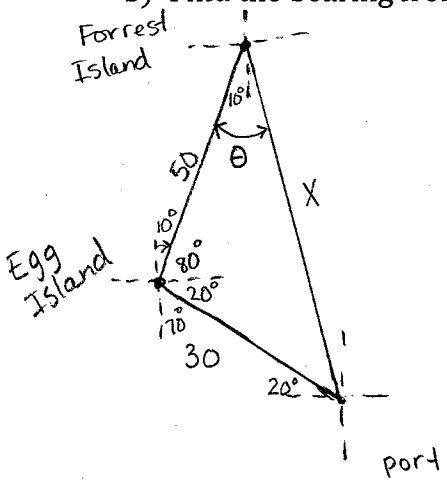
$$\theta = 6.63°$$

Bearing N 26.63° E

10. A fisherman leaves his home port and heads in the direction of N70°W. He travels 30 miles and reaches Egg Island. The next day he sails N10°E for 50 mi, reaching Forrest Island.

a) Find the distance between the fisherman's home port and Forrest Island. 62.62 mi

b) Find the bearing from Forrest Island back to his home port. S 18.2° E



$$x^2 = 30^2 + 50^2 - 2(30)(50)\cos 100°$$

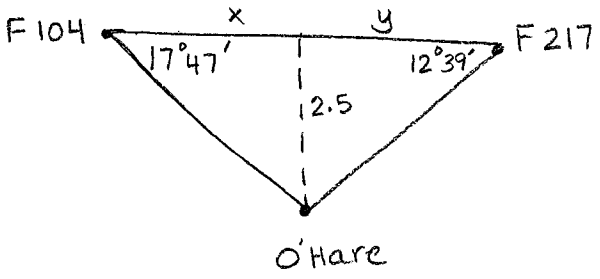
$$x = 62.62 \text{ mi}$$

$$\frac{\sin\theta}{30} = \frac{\sin 100°}{62.62}$$

$$\theta = 28.2°$$

S 18.2° E

11. Flights 104 and 217 are both approaching O'Hare International Airport from directions directly opposite one another and at an altitude of 2.5 miles. The pilot on flight 104 reports an angle of depression of 17º47' to the tower, and the pilot on flight 217 reports an angle of depression of 12º39' to the tower. Calculate the distance between the planes. 18.93 miles



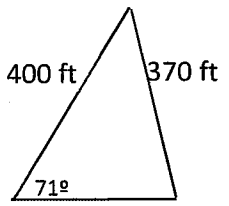
$\tan 17°47 = \frac{2.5}{x}$

$x = 7.79$ miles

$\tan 12°39' = \frac{2.5}{y}$

$y = 11.14$ miles

12. A real estate agent has just take a trigonometry class at the local community college. She is considering purchasing a piece of property and is waiting for the surveyor's report before closing the deal. If the surveyor submits a drawing as in the figure below, explain why the agent will reject the sale.

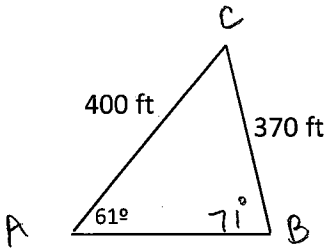


$\frac{\sin B}{400} = \frac{\sin 71°}{370}$

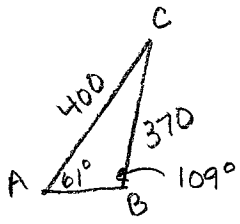
$\angle B$ = domain error

Doesn't even make a triangle

13. The surveyor admits to his mistake and revises his drawing as in the next figure. This time the real estate agent refuses to complete the deal until additional information is supplied. What additional information is the real estate agent looking for to complete her knowledge about the parcel of land?



Case I



or case II

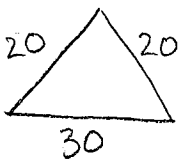
case I

$\frac{\sin B}{400} = \frac{\sin 61°}{370}$

$\angle B = 71°$

She needs to know the measurement of $\angle B$.

14. A portion of a barn, in the shape of an isosceles triangle, must be painted. The base of the triangle measures 30 feet long and the legs measure 20 feet each. A can of weatherproofing paint will cover 50 square feet of area. What is the minimum number of cans needed to cover this triangular portion? Show your work.



$S = \left(\frac{20+20+30}{3}\right) = 35$

$A = \sqrt{35(35-20)(35-20)(35-30)}$

$A = 198.43\ ft^2$

4 cans