Precalculus

Section 7.1&7.4 review

Name Key

Period

Find the partial fraction decomposition.

1. $\dfrac{x+14}{x^2-2x-8} = \dfrac{A}{x-4} + \dfrac{B}{x+2}$

$(x-4)(x+2)$

$\Rightarrow = A(x+2) + B(x-4)$

$= Ax + 2A + Bx - 4B$

$x + 14 = x(A+B) + 2A - 4B$

$A + B = 1 \qquad A = 3$

$2A - 4B = 14 \qquad B = -2$

$\boxed{\dfrac{3}{x-4} + \dfrac{-2}{x+2}}$

2. $\dfrac{x}{8x^2-10x+3} = \dfrac{A}{4x-3} + \dfrac{B}{2x-1}$

$(4x-3)(2x-1)$

$\Rightarrow = A(2x-1) + B(4x-3)$

$= 2Ax - A + 4Bx - 3B$

$x = x(2A+4B) - A - 3B$

$2A + 4B = 1 \qquad A = 3/2$

$-A - 3B = 0 \qquad B = -1/2$

$\boxed{\dfrac{\frac{3}{2}}{4x-3} + \dfrac{-\frac{1}{2}}{2x-1}}$

3. $\dfrac{9x^2-9x+6}{2x^3-x^2-8x+4}$

$x^2(2x-1) - 4(2x-1)$

$(2x-1)(x^2-4)$

$(2x-1)(x+2)(x-2)$

$\dfrac{A}{2x-1} + \dfrac{B}{x+2} + \dfrac{C}{x-2} \Rightarrow A(x+2)(x-2) + B(2x-1)(x-2) + C(2x-1)(x+2)$

$= A(x^2-4) + B(2x^2-5x+2) + C(2x^2+3x-2)$

$= Ax^2 - 4A + 2Bx^2 - \cancel{5Bx} + 2B + 2Cx^2 + \cancel{3Cx} - 2C$

$9x^2 - 9x + 6 = x^2(A+2B+2C) + x(-5B+3C) - 4A + 2B - 2C$

$A + 2B + 2C = 9 \qquad A = -1$

$-5B + 3C = -9 \qquad B = 3$

$-4A + 2B - 2C = 6 \qquad C = 2$

$\boxed{\dfrac{-1}{2x-1} + \dfrac{3}{x+2} + \dfrac{2}{x-2}}$

4. $\dfrac{4x^2-x-2}{x^4+2x^3} = \dfrac{A}{x^3} + \dfrac{B}{x^2} + \dfrac{C}{x} + \dfrac{D}{x+2}$

$x^3(x+2)$

$\Rightarrow A(x+2) + B(x)(x+2) + C(x^2)(x+2) + D(x^3)$

$= \cancel{Ax} + 2A + Bx^2 + \cancel{2Bx} + Cx^3 + \cancel{2Cx^2} + Dx^3$

$4x^2 - x - 2 = x^3(C+D) + x^2(B+2C) + x(A+2B) + 2A$

$C + D = 0 \qquad A = -1$

$B + 2C = 4 \qquad B = 0$

$A + 2B = -1 \qquad C = 2$

$2A = -2 \qquad D = -2$

$\boxed{\dfrac{-1}{x^3} + \dfrac{2}{x} \quad \dfrac{-2}{x+2}}$

5. $\dfrac{x^2}{x^3-4x^2+5x-2}$

Graph on Calculator

$(x-1)^2(x-2)$ repeats

$\Rightarrow \dfrac{A}{(x-1)} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2}$

$1 \mid 1 \quad -4 \quad 5 \quad -2$

$\downarrow \quad 1 \quad -3 \quad 2$

$1 \quad -3 \quad 2 \quad 0$

$(x-2)(x-1)$

$= A(x-1)(x-2) + B(x-2) + C(x-1)^2$

$= A(x^2-3x+2) + Bx - 2B + C(x^2-2x+1)$

$= Ax^2 - 3Ax + 2A + Bx - 2B + Cx^2 - 2Cx + C$

$A + C = 1 \qquad A = -3$

$-3A + B - 2C = 0 \qquad B = -1$

$2A - 2B + C = 0 \qquad C = 4$

$\boxed{\dfrac{-3}{x-1} + \dfrac{-1}{(x-1)^2} + \dfrac{4}{x-2}}$

6. $\frac{x+3}{x^3-19x+30}$

graph on calculator

$(x+5)(x-2)(x-3)$

$\Rightarrow = \frac{A}{x+5} + \frac{B}{x-2} + \frac{C}{x-3}$

$= A(x-2)(x-3) + B(x+5)(x-3) + C(x+5)(x-2)$

$= A(x^2-5x+6) + B(x^2+2x-15) + C(x^2+3x-10)$

$x+3 = Ax^2 - 5Ax + 6A + Bx^2 + 2Bx - 15B + Cx^2 + 3Cx - 10C$

$A+B+C=0$

$-5A+2B+3C=1$

$6A-15B-10C=3$

$A = \frac{-1}{28}$

$B = -5/7$

$C = 3/4$

$$\boxed{\frac{\frac{-1}{28}}{x+5} + \frac{-5/7}{x-2} + \frac{3/4}{x-3}}$$

7. $\frac{x^5-2x^4+x^3+x+5}{x^3-2x^2+x-2}$

$x^2(x-2)+(x-2)$

$(x-2)(x^2+1)$

SKIP

Solve by substitution.

8. $x^2+y^2=25$

$x+2y=5$ $\quad x = 5-2y$

$(5-2y)^2 + y^2 = 25$

$25 - 20y + 4y^2 + y^2 = 25$

$5y^2 - 20y = 0$

$5y(y-4) = 0$

$y=0$ or $y=4$

$(5, 0)$

$(-3, 4)$

Solve by elimination.

9. $^{-2}(4x-3y) = (11)^{-2}$

$8x+4y=122$

$-8x+6y=-22$

$10y = 100$

$y = 10$

$4x - 3(10) = 11$

$4x = 41$

$x = 41/4$

$(41/4, 10)$

Solve by graphing.

10. $x^2+y^2=25$

$x+3y=2$

$y = \pm\sqrt{25-x^2}$

$\frac{3y}{3} = \frac{-x}{3} + \frac{2}{3}$

$y = -1/3x + 2/3$

$(-4.51, 2.17)$

$(4.91, -.97)$

Name Key ID: 1

Conics review

Date_____ Period____

Classify each conic section. (Circle, ellipse, parabola, hyperbola)

1) $-9x^2 + 16y^2 + 18x - 153 = 0$
hyperbola

2) $4x^2 + 9y^2 + 8x - 72y + 112 = 0$
ellipse

3) $x^2 + y^2 - 8x + 8y + 31 = 0$
circle

4) $9x^2 + 25y^2 + 36x - 150y + 36 = 0$
ellipse

5) $-9x^2 + 16y^2 - 64y - 80 = 0$
hyperbola

6) $25x^2 - 16y^2 + 50x - 375 = 0$
hyperbola

7) $4x^2 + y^2 - 4y - 16 = 0$
ellipse

8) $2y^2 + 7x - 8y + 43 = 0$
parabola

9) $x^2 + y^2 + 6x + 6y + 9 = 0$
circle

10) $-y^2 + x - 10y - 27 = 0$
parabola

11) $-2x^2 + y = 0$
parabola

12) $49x^2 + 9y^2 - 98x - 392 = 0$
ellipse

13) $16x^2 - 9y^2 - 32x - 18y - 137 = 0$
hyperbola

14) $x^2 + 4y^2 + 32y + 28 = 0$
ellipse

15) $2x^2 + 2y^2 + 14x + 6y + 11 = 0$
circle

16) $2x^2 + 2y^2 + 14x - 10y + 19 = 0$
circle

Classify each conic section and sketch its graph. Identify the Vertices, foci and eccentricty where it applies.

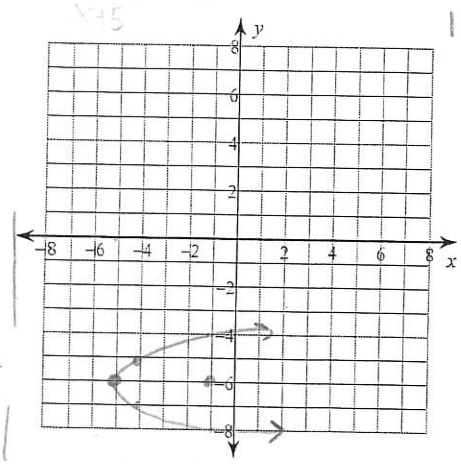
parabola

17) $x = (y+6)^2 - 5$

$a = \frac{1}{4}p$

$1 = 1/4\,p$

$p = 4$



$x = -9$ directrix

$(-1,-6)$ focus

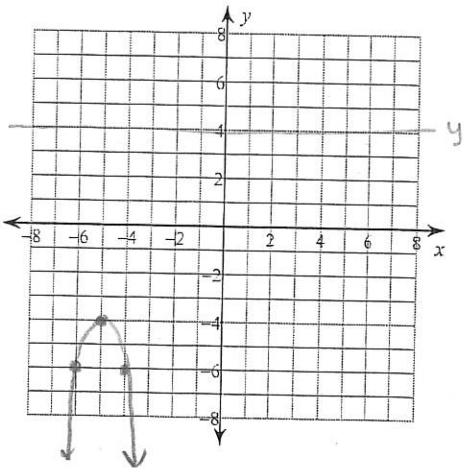
parabola

18) $y = -2(x+5)^2 - 4$

$a = \frac{1}{4}p$

$-2 = \frac{1}{4}p$

$p = -8$

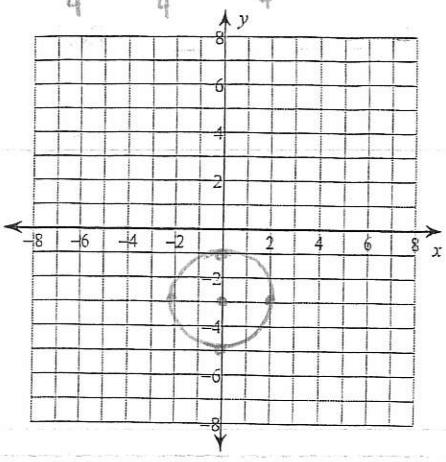


$y = 4$ directrix

Focus $(-5, -12)$

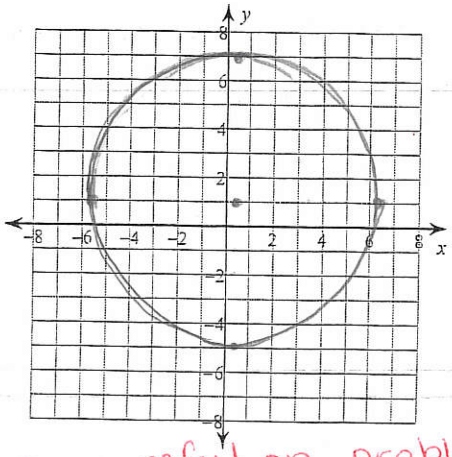
circle

19) $\frac{x^2}{4} + \frac{(y+3)^2}{4} = \frac{4}{4}$ c: (0, -3)



circle

20) $\left(x - \frac{1}{2}\right)^2 + (y-1)^2 = 36$ c: $(\frac{1}{2}, 1)$



Be careful on problem numbering

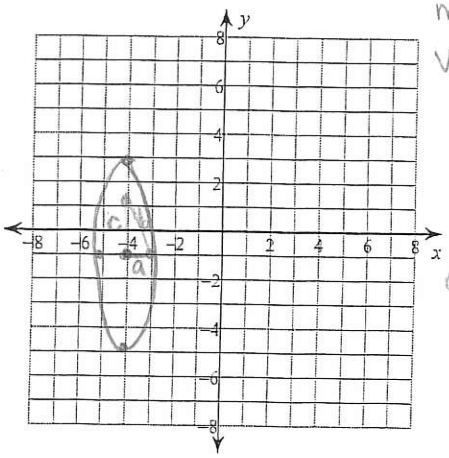
21) $(x+4)^2 + \frac{(y+1)^2}{16} = 1$

ellipse

major axis: $x = -4$

minor axis: $y = -1$

Vertices: $(-4, 3)$
$(-4, -5)$
$(-3, -1)$
$(-5, -1)$



$a^2 + c^2 = b^2$

$1 + c^2 = 4^2$

$c = \sqrt{15}$

Foci
$(-4, -1+\sqrt{15})$
$(-4, -1-\sqrt{15})$

eccentricity $= e = \frac{c}{a}$

$e = \frac{\sqrt{15}}{4}$

22) $\frac{x^2}{40} + \frac{(y-3)^2}{5} = 1$

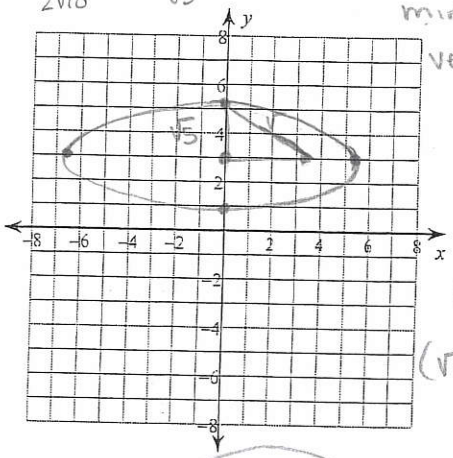
$2\sqrt{10}$ $\sqrt{5}$

ellipse

major axis: $y = 3$

minor axis: $x = 0$

vertices:
$(0, 3+\sqrt{5})$
$(0, 3-\sqrt{5})$
$(-2\sqrt{10}, 3)$
$(2\sqrt{10}, 3)$



Foci

$(\sqrt{5})^2 + b^2 = (\sqrt{40})^2$

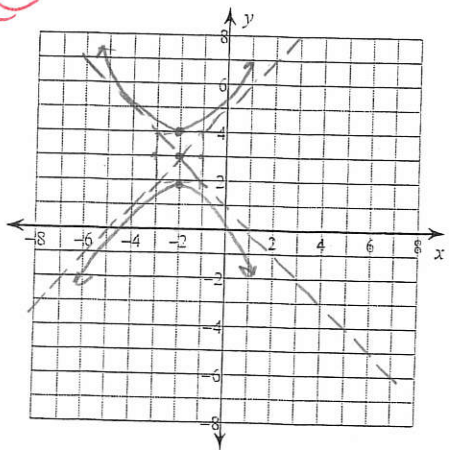
$5 + b^2 = 40$

$b = \sqrt{35}$

$e = \frac{\sqrt{35}}{\sqrt{40}}$

Foci: $(\sqrt{35}, 3)$
$(-\sqrt{35}, 3)$

23) $(y-3)^2 - (x+2)^2 = 1$ hyperbola



24) $(x+4)^2 - \frac{y^2}{25} = 1$ hyperbola