Honors Math II
Chapter 5 Review

Name Key
Date_____ Period_____

Write a polynomial function of least degree that has rational coefficients, a leading coefficient of 1 and the given zeros.

1. $1, 2, 3$

$(x-1)(x-2)(x-3)$
$(x-1)(x^2-3x-2x+6)$
$(x-1)(x^2-5x+6)$
x^3-5x^2+6x
$\quad -x^2+5x-6$
$x^3-6x^2+11x-6$

2. $-2, 1, 3$

$(x+2)(x-1)(x-3)$
$(x+2)(x^2-3x-1x+3)$
$(x+2)(x^2-4x+3)$
x^3-4x^2+3x
$\quad +2x^2-8x+6$
x^3-2x^2-5x+6

3. $\sqrt{3}, -\sqrt{3}$

$(x-\sqrt{3})(x+\sqrt{3})$
$x^2+\sqrt{3}x-\sqrt{3}x-3$
x^2-3

4. $5i, -5i$

$(x-5i)(x+5i)$
$x^2+5ix-5ix-25i^2$
x^2+25

5. $4, \sqrt{5}, -\sqrt{5}$

$(x-4)(x-\sqrt{5})(x+\sqrt{5})$
$(x-4)(x^2-5)$
$x^3-5x-4x^2+20$
$x^3-4x^2-5x+20$

6. $-3, 1, 6$

$(x+3)(x-1)(x-6)$
$(x+3)(x^2-7x+6)$
x^3-7x^2+6x
$\quad +3x^2-21x+18$
$x^3-4x^2-15x+18$

7. $2, -2$

$(x-2)(x+2)$
x^2-4

8. $4+i, 4-i$

$(x-4-i)(x-4+i)$
$x^2-4x+ix$
$\quad -4x \quad +16-4i$
$\quad -ix \quad +4i \quad -i^2$
$x^2-8x+16+1$
$x^2-8x+17$

9. $3+2i, 3-2i$

$(x-3-2i)(x-3+2i)$
$x^2-3x+2ix$
$\quad -3x \quad +9-6i$
$\quad -2ix \quad +6i \quad -4i^2$
$x^2-6x \quad +9 \quad +4$
$x^2-6x+13$

10. $7i, -7i$

$(x-7i)(x+7i)$
x^2-49i^2
x^2+49

Divide using long division.

11. $5x^4 + 2x^3 - 9x + 12 \div x^2 - 3x + 4$

$$
\begin{array}{r}
5x^2 + 17x + 31 \\
x^2 - 3x + 4 \overline{\big) 5x^4 + 2x^3 + 0x^2 - 9x + 12} \\
-5x^4 + 15x^3 - 20x^2 \\
\hline
17x^3 - 20x^2 - 9x \\
-17x^3 + 51x^2 - 68x \\
\hline
31x^2 - 77x + 12 \\
-31x^2 + 93x - 124 \\
\hline
16x - 112
\end{array}
\quad + \frac{16x - 112}{x^2 - 3x + 4}
$$

12. $x^3 + 11x^2 + 25x - 21 \div x + 7$

$$
\begin{array}{r}
x^2 + 4x - 3 \\
x + 7 \overline{\big) x^3 + 11x^2 + 25x - 21} \\
-x^3 - 7x^2 \\
\hline
4x^2 + 25x \\
-4x^2 - 28x \\
\hline
-3x - 21 \\
+3x + 21 \\
\hline
0
\end{array}
$$

Divide using synthetic division.

13. $2x^4 - 11x^3 + 15x^2 + 6x - 18 \div x - 3$

$$
\begin{array}{r|rrrrr}
3 & 2 & -11 & 15 & 6 & -18 \\
 & & 6 & -15 & 0 & 18 \\
\hline
 & 2 & -5 & 0 & 6 & 0
\end{array}
$$

$\boxed{2x^3 - 5x^2 + 6}$

14. $x^4 - 6x^3 - 40x + 33 \div x - 7$

$$
\begin{array}{r|rrrrr}
7 & 1 & -6 & 0 & -40 & 33 \\
 & & 7 & 7 & 49 & 63 \\
\hline
 & 1 & 1 & 7 & 9 & 96
\end{array}
$$

$x^3 + x^2 + 7x + 9 + \frac{96}{x-7}$

Factor completely.

15. $9y^2 - 64$ $(3y+8)(3y-8)$

16. $5x^3 + 5x^2 + 30x$

$5x(x^2 + x + 6)$

17. $x^3 + 8y^3$ $(x+2y)(x^2 - 2xy + 4y^2)$

18. $5x^4 + 10x^2 - 15$

$5(x^4 + 2x^2 - 3)$

$5(x^2+3)(x^2-1) \Rightarrow 5(x^2+3)(x-1)(x+1)$

19. $2x^3 - 3x^2 + 4x - 6$

$x^2(2x-3) + 2(2x-3)$

$(2x-3)(x^2+2)$

20. $x^3 + 3x^2 - 4x - 12$

$x^2(x+3) - 4(x+3)$

$(x+3)(x^2-4)$

$(x+3)(x-2)(x+2)$

21. $64x^3 - 27y^3$ $(4x-3y)(16x^2 + 12xy + 9y^2)$

22. $x^4 - 13x^2 + 36$ $(x^2-9)(x^2-4)$

$(x+3)(x-3)(x+2)(x-2)$

23. $15x^3 + 14x^2 - 3x - 2$ if (x + 1) is one of the factors.

$$
\begin{array}{r|rrrr}
-1 & 15 & 14 & -3 & -2 \\
 & & -15 & 1 & 2 \\
\hline
 & 15 & -1 & -2 & 0
\end{array}
$$

$(x+1)(15x^2 - x - 2)$

$(x+1)(3x+1)(5x-2)$

24. $x^3 - x^2 - 8x + 12$ if (x + 3) is one of the factors.

$$
\begin{array}{r|rrrr}
-3 & 1 & -1 & -8 & 12 \\
 & & -3 & 12 & -12 \\
\hline
 & 1 & -4 & 4 & 0
\end{array}
$$

$(x+3)(x^2 - 4x + 4)$

$(x+3)(x-2)(x-2) \Rightarrow (x+3)(x-2)^2$

Solve.

25. $x^4 - 9x^2 + 14 = 0$

$(x^2-7)(x^2-2)=0$

$x^2-7=0 \quad x^2-2=0$

$x^2=7 \quad x^2=2$

$x=\pm\sqrt{7} \quad x=\pm\sqrt{2}$

26. $10x^4 - 160 = 0$

$10(x^4-16)=0$

$10(x^2+4)(x^2-4)=0$

$x^2+4=0 \quad x^2-4=0$

$x^2=-4 \quad x^2=4$

$x=\pm\sqrt{-4} \quad x=\pm\sqrt{4}$

$x=\pm 2i \quad x=\pm 2$

27. $6x^2 + 11x - 35 = 0$

$(3x-5)(2x+7)=0$

$3x-5=0 \quad 2x+7=0$

$3x=5 \quad 2x=-7$

$x=5/3 \quad x=-\frac{7}{2}$

28. $12x^2 - 21x - 45 = 0$

$3(4x^2-7x-15)=0$

$3(4x+5)(x-3)=0$

$4x+5=0 \quad x-3=0$

$4x=-5 \quad x=3$

$x=\frac{-5}{4}$

Write the end behavior for the polynomial function.

29. $g(x) = 2x^3 - 3x + 1$

$x \to -\infty \; f(x) \to -\infty$

$x \to \infty \; f(x) \to \infty$

30. $f(x) = -3x^7 + 5x^4 - 2x + 4$

$x \to -\infty \; f(x) \to \infty$

$x \to \infty \; f(x) \to -\infty$

31. $h(x) = x^4 - 4x + 2$

$x \to -\infty \; f(x) \to \infty$

$x \to \infty \; f(x) \to \infty$

32. $f(x) = -6x^8 - 25$

$x \to -\infty \; f(x) \to -\infty$

$x \to \infty \; f(x) \to -\infty$

Decide whether the function is a polynomial function. If so, state its degree, type (constant, linear, quadratic, etc.), and leading coefficient.

33. $f(x) = 78$ yes

Constant function

degree 0

34. $g(x) = -2x^3 - 2x^2 + 3$

L.C $\Rightarrow -2$

cubic function

Degree 3

35. $g(x) = 4x^2 - x^{-2} + 3$

Not a polynomial

36. $f(x) = -8 + 3x + x^4 - x^2$ $\quad x^4 - x^2 + 3x - 8$

L.C. $\Rightarrow 1$

quarctic

Degree 4

Perform the indicated operation.

37. $(x^3 + x^2 - 6) + (2x^2 + 4x + 8)$

$x^3 - x^2 - 4x + 2$

38. $(-3x^2 + 4x - 10) + (x^2 - 9x + 15)$

$-2x^2 - 5x + 5$

39. Write a cubic function whose graph passes through the points.

$(3,0), (-3,0), (4,0), (0,36)$

↑ y-intercept

$a = 1$

$y = a(x-3)(x+3)(x-4)$

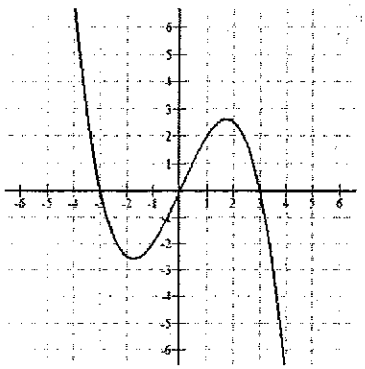
$y = a(x^2-9)(x-4)$

$y = x^3 - 4x^2 - 9x + 36$

↑ y-intercept

$\boxed{x^3 - 4x^2 - 9x + 36}$

40. Write a cubic function whose graph is shown.



$y = a(x+3)(x-3)(x+0)$

$y = a(x^2-9)(x)$

$y = a(x^3-9x)$

$y = -\frac{1}{4}(x^3-9x)$

$y = -\frac{1}{4}x^3 + \frac{9}{4}x$

Because the y-intercept is zero you can't tell what "a" equals.

Pick another known point on the grid. It is hard to tell but $(4,-7)$ is a point you can use.

$-7 = a(4+3)(4-3)(4)$

$-7 = 28a$

$a = -1/4$

Simplify.

41. $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

42. $\left(\frac{2}{3^{-2}}\right)^2 = \frac{4}{3^{-4}} = 4 \cdot 3^4 = 324$

43. $(x^4y^{-2})(x^{-3}y^8) = xy^6$

44. $(a^2b^{-5})^{-3} = a^{-6}b^{15} = \frac{b^{15}}{a^6}$

45. $(-3c^3d^4e^6)^2$ $(-3)^2c^6d^8e^{12}$

$9c^6d^8e^{12}$

46. $\frac{12x^{-3}}{y^4} \cdot \frac{(y^{-2}x^2)^{-2}}{15x^{-2}} = \frac{12}{y^4x^3} \cdot \frac{y^4x^{-4}}{15x^{-2}} = \frac{4y^4x^{-4}}{5y^4x}$

$\boxed{\frac{4}{5x^5}}$

47. $\frac{x^3y^7}{x^{-4}y^0} = x^7y^7$

48. $\frac{c^3d^{-2}}{c^5d^{-1}} = \frac{c^3d}{c^5d^2} = \frac{1}{c^2d}$