Precalculus
Unit 7 Test Review

Name Key
Period_____ Date_____

Solve by elimination.

1. $(3x + 3y = -15)3$
$5x - 9y = 3$

$9x + 9y = -45$
$5x - 9y = 3$
$14x = -42$
$x = -3$
$3(-3) + 3y = -15$
$3y = -6$
$y = -2$

$\boxed{(-3, -2)}$

2. $(3x - 6y = 9)4$
$(-4x + 7y = -16)3$

$12x - 24y = 36$
$-12x + 21y = -48$
$-3y = -12$
$y = 4$
$3x - 6(4) = 9$
$3x = 33$
$x = 11$

$\boxed{(11, 4)}$

3. $12x - 3y = -9$
$(-4x + y = 3)3$

$12x - 3y = -9$
$-12x + 3y = 9$
$0 = 0$

$\boxed{\text{infinite solutions}}$

4. $(4x - 5y = 13)2$
$(6x + 2y = 48)5$

$8x - 10y = 26$
$30x + 10y = 240$
$38x = 266$
$x = 7$
$4(7) - 5y = 13$
$-5y = -15$
$y = 3$

$\boxed{(7, 3)}$

Solve the system by graphing.

5. $y = x^3 - 4x$
$4 = x - 2y$

$\frac{4}{-2} = \frac{-2y}{-2}$
$y = \frac{1}{2}x - 2$

$y = x^3 - 4x$
$y = \frac{1}{2}x - 2$

$\boxed{(-2.31, -3.15)\ (.467, -1.766)\ (1.84, -1.075)}$

6. $y = 1 + 2x - x^2$
$y = 1 - x$

$\boxed{(0, 1)\ (3, -2)}$ about

Solve the system by substitution.

7. $x = y + 3$
$x - y^2 = 3y$

$y + 3 - y^2 = 3y$
$-y^2 - 2y + 3 = 0$
$y^2 + 2y - 3 = 0$
$(y + 3)(y - 1) = 0$
$y = -3 \quad y = 1$

$\boxed{(0, -3)\ (4, 1)}$

8. $y = x^2$
$y - 25 = 0$

$x^2 - 25 = 0$
$(x + 5)(x - 5) = 0$
$x = -5 \quad x = 5$

$\boxed{(5, 25)\ (-5, 25)}$

9. $y = x^2 - 5x + 6$
$y = -x + 6$

$-x + 6 = x^2 - 5x + 6$
$0 = x^2 - 4x$
$0 = x(x - 4)$
$x = 0 \quad x = 4$

$\boxed{(0, 6)\ (4, 2)}$

10. $y = x^3 + x^2$
$y = 2x^2$

$2x^2 = x^3 + x^2$
$0 = x^3 - x^2$
$0 = x^2(x - 1)$
$x = 0 \quad x = 1$

$\boxed{(0, 0)\ (1, 2)}$

Simplify the following rational expressions.

LCD: $15x^2(x-1)$

11. $\frac{2x-1}{3} + \frac{-7}{x-3}$

$\frac{(2x-1)(x-3)+-7(3)}{3(x-3)} =$

$\frac{2x^2-6x-x+3-21}{3(x-3)} = \boxed{\frac{2x^2-7x-17}{3(x-3)}}$

12. $\frac{7}{3x} + \frac{-2x+1}{5(x-1)} + \frac{4}{15x^2}$

$7(5x)(x-1)+(-2x-1)(3x^2)+4(x-1)$

$35x(x-1)-6x^3-3x^2+4x-4$

$35x^2-35x-6x^3-3x^2+4x-4$

$\boxed{\frac{-6x^3+32x^2-31x-4}{15x^2(x-1)}}$

13. $\frac{3x+4}{x^2-9} + \frac{-x+1}{x-3}$

$(x+3)(x-3)$

$3x+4+(-x-1)(x+3)$

$3x+4-x^2-3x-x-3$

$\boxed{\frac{-x^2-x+1}{(x+3)(x-3)}}$

14. $\frac{3x-2}{x+2} + \frac{x+1}{x-3}$

$(3x-2)(x-3)+(x+1)(x+2)$

$3x^2-9x-2x+6+x^2+2x+x+2$

$\boxed{\frac{4x^2-8x+8}{(x+2)(x-3)}}$

Decompose the fraction into partial fractions:

15. $\frac{4x-11}{2x^2-x-3} = \frac{A}{2x-3} + \frac{B}{x+1}$

$(2x-3)(x+1)$

$\boxed{\frac{-2}{2x-3} + \frac{3}{x+1}}$

$4x-11 = A(x+1)+B(2x-3)$

$4x-11 = Ax+A+2Bx-3B$

$= Ax+2Bx+A-3B$

$4x-11 = x(A+2B)+A-3B$

$A+2B=4$

$-A+3B=+11$

$5B=15$

$B=3$

$A+2(3)=4$

$A=-2$

16. $\dfrac{x^2-4x+7}{(x+1)(x^2-2x+3)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2-2x+3}$

$$\boxed{\frac{2}{x+1} + \frac{-x+1}{x^2-2x+3}}$$

$$x^2-4x+7 = A(x^2-2x+3) + (Bx+C)(x+1)$$

$$= Ax^2-2Ax+3A + Bx^2+Bx+Cx+C$$

$$= Ax^2+Bx^2 - 2Ax+Bx+Cx+3A+C$$

$$x^2-4x+7 = x^2(A+B) + x(-2A+B+C) + 3A+C$$

$A+B=1$

$-2A+B+C=-4$

$3A+C=7$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ -2 & 1 & 1 & -4 \\ 3 & 0 & 1 & 7 \end{bmatrix}$$

$A=2$

$B=-1$

$C=1$

17. $\dfrac{x^2+1}{x(x-1)^3} = \dfrac{A}{x} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2} + \dfrac{D}{(x-1)^3}$

$\uparrow$ repeated linear

$$\boxed{\frac{-1}{x} + \frac{1}{x-1} + \frac{2}{(x-1)^3}}$$

$$= A(x-1)^3 + Bx(x-1)^2 + Cx(x-1) + D(x)$$

$$= A(x-1)(x^2-2x+1) + Bx(x^2-2x+1) + Cx^2 - Cx + Dx$$

$$= A(x^3-2x^2+x-x^2+2x-1) + Bx^3-2Bx^2+Bx+Cx^2-Cx+Dx$$

$$x^2+1 = \cancel{Ax^3} - \cancel{3Ax^2} + \cancel{3Ax} - A + \cancel{Bx^3} - \cancel{2Bx^2} + \cancel{Bx} + \cancel{Cx^2} - \cancel{Cx} + \cancel{Dx}$$

$$x^2+1 = x^3(A+B) + x^2(-3A-2B+C) + x(3A+B-C+D) - A$$

$A+B=0$

$-3A-2B+C=1$

$3A+B-C+D=0$

$-A=1$

$A=-1$

$B=1$

$C=0$

$-3(-1)-2(1)+C=1$

$3-2+C=1$

$1+C=1$

$C=0$

$3(-1)+(1)-(0)+D=0$

$-3+1+D=0$

$-2+D=0$

$D=2$

18. $\dfrac{x-3}{x^3+3x} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+3}$

$x(x^2+3)$

$$\boxed{\frac{-1}{x} + \frac{x+1}{x^2+3}}$$

$$x-3 = A(x^2+3) + x(Bx+C)$$

$$= Ax^2+3A+Bx^2+Cx$$

$$= Ax^2+Bx^2+Cx+3A$$

$$x-3 = x^2(A+B)+Cx+3A$$

$A+B=0$

$C=1$

$3A=-3$

$A=-1$

$B=1$

$C=1$

19. $\dfrac{12x+12}{x^2-4x} = \dfrac{A}{x} + \dfrac{B}{x-4}$

$x(x-4)$

$$\boxed{\frac{-3}{x} + \frac{15}{x-4}}$$

$$12x+12 = A(x-4)+Bx$$

$$12x+12 = Ax-4A+Bx$$

$$12x+12 = x(A+B)-4A$$

$A+B=12$

$-4A=12$

$A=-3$

$B=15$